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Research article

Uncertainty of Uptake in Speech Acts

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Abstract:
When something is said by an agent, addressees and other agents in the audience may sometimes be uncertain what specific illocutionary act the agent intends to perform even if what is said and the context in which it is said are perfectly clear. Yet which illocutionary act is performed can make a great difference in what the addressees should do in response to the utterance. The well-known method for analyzing how the different (un)certainties different agents have about what has been done affect the outcome is the method of the product update by action models. This method is used in developing various systems in DEL (Dynamic Epistemic Logic) that deal with acts that affect epistemic states of agents. Illocutionary acts, however, usually affect deontic aspects of the situations in which they are performed. We will define a deontic version of the method of product update by importing ideas from dynamic deontic logics that deal with acts of commanding and acts of requesting. We then show (1) that the deontic version of product update works when uptake (the understanding of the force and the content of an illocutionary act) is secured, but (2) that it doesn't work when uptake is not secured, and (3) that a twist is needed to represent what is going on in such a case.

Keywords:
Uptake, Uncertainty, Action model, Product update, Dynamic deontic logic, Deontic product update, Illocutionary act

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1. Introduction

When something is said by an agent, addressees and other agents in the audience may sometimes be uncertain what specific illocutionary act the agent intends to perform even if what is said and the context in which it is said are perfectly clear. Yet which illocutionary act is performed can make a great difference in what the addressees should do in response to the utterance. Consider acts of commanding and requesting. As Searle and Vanderveken have pointed out, a request "allows for the possibility of refusal" (Searle and Vanderveken 1985, p. 199), but a command "commits the speaker to not giving him [= the commandee (the present author's clarification)] the option of refusal" (op. cit., 201). Since there are sentences that can be used in performing both of these two kinds of acts, however, the audience may sometimes be uncertain whether a command is given or a request is made. As Austin has pointed out,

> Unless a certain effect is achieved, the illocutionary act will not have been happily, successfully performed. This is not to say that the illocutionary act is the achieving of a certain effect. I cannot be said to have warned an audience unless it hears what I say and takes what I say in a certain sense. An effect must be achieved on the audience if the illocutionary act is to be carried out. · · · Generally the effect amounts to bringing about the understanding of the meaning and the force of the locution. So the performance of an illocutionary act involves the securing of *uptake* (Austin 1955, pp. 116–117).

Our purpose in this paper is to examine what will happen if an agent uses a locution that can be used to perform two or more illocutionary acts. Our examples are those in which an agent uses a locution that can be used in both giving a command and making a request.

The remaining part of this paper is structured as follows. In Section 2, we present a scenario in which uptake is not secured. This is the most difficult scenario we will analyze in this paper. Our analysis will be given in Section 5. Before that, in Section 3, we briefly review the method often used for analyzing situations where the different (un)certainties different agents have about what has been done are included. It is the method of the product update introduced in Baltag, Moss, & Solecki (1998). We briefly review how this method works in DEL (Dynamic Epistemic Logic).

Then, in Sections 4 and 5, we examine how a similar method can be applied to examples in which the question of whether a command is given or a request is made may be raised. Since we need to define how acts of commanding and requesting change situations in order to define a deontic version of product update that can deal with such cases, we import some ideas from DMEDL (Dynamified Multi-agent Epistemic Deontic Logic) that deals with acts of commanding and requesting developed in Yamada (2011, 2016).[1]

As it happens, the analysis of acts of requesting presented in Yamada (2011) and that presented in Yamada (2016) are different. We follow the version of the semantics of DMEDL given in Yamada (2016) in this paper. It is based on what we call "the two-option analysis of requesting," because the other version presented in Yamada (2011), which is based on what we call "the three-option analysis of requesting," is not suitable for differentiating the effects of acts of requesting from those of acts of commanding, as is shown in Appendix.

In Section 4, we define a deontic version of action model logic DAM^{-} based on the two-option analysis. We then apply it to an example in which the addressee correctly understands that a request is made but some other agent wonders whether a request is made or a command is given. We show that the deontic product update works nicely for this example.

Then, in Section 5, we examine whether or not the deontic action model logic can be applied to the scenario in which uptake is not secured and show that some twist is needed in order to represent what has happened in it. We show what problem needs to be addressed and how it can be handled when we take the necessity of the securing of uptake for performing illocutionary acts into consideration.

And finally, in Section 6, we conclude the paper with a brief overview of how the deontic product update and the epistemic product update can be used to represent different (un)certainties about different aspects of speech acts.

2. A Scenario

Consider the following example.

Example 1 Suppose an agent *a* receives a phone call from *c*, who is the guru of the political group of which *a* is a member. The guru says to *a*, "There will be an important demonstration

1 I presented the basic idea of a deontic version of product update in the talk titled "Product Update for Dynamified Deontic Logic of Speech Acts" in the 15th Congress of Logic, Methodology and Philosophy of Science (University of Helsinki, Finland, 3–8 August 2015). A slightly longer version of the talk was also given with the same title in TLC 2015 (The 3rd Tsinghua Logic Colloquium, Tsinghua University, Beijing, 20–21 October 2015). I thank the participants of these two meetings for their stimulating comments and discussions. Since then, various revisions have been incorporated. Especially, the definition of the action model is changed, the examples are replaced, the syntax and the semantics are explicitly defined, and the proposal of applying epistemic product update by M^{G} and deontic product update by M^{C} to the example in which uptake is not secured are added. As the manuscript of this paper was anonymized for blind review, this footnote is added after it was accepted for publication.

in Tokyo on August 1st next year. I would be happy if you could lead our group in that demonstration." Unfortunately, an international one day conference on logic will be held in São Paulo on that day, and *a* has already promised his former student who is organizing the conference that he will give an invited lecture in it. The following exchange of words follows.

a: Ma'am, is it a command or a request? If it is a request, I'm very sorry, but I'd like to decline.

c: Then, it's a command.

a: Yes, Ma'am. □

Let *r* be the proposition that *a* will lead their group in the demonstration in Tokyo on August 1st next year. The locution used by the guru, namely "I would be happy if you could lead our group in that demonstration" can be used to request *a* to see to it that *r*, but it can also be used to command *a* to do so. Moreover, *c* has suitable authority for commanding *a* to do so, but she is also in a position to request *a* to do so. Thus, *a* may well wonder whether *c* has commanded or requested.[2]

If *a* joins the demonstration in Tokyo, *a* will not be able to keep the promise he has given to his former student *b*. Thus, he was not ready to join the demonstration when the guru said to him "I would be happy if you could lead our group in that demonstration," but he was not ready to disobey her either. So, he decided to obey her when she said to him, "Then, it's a command." So, he now has to withdraw his earlier promise given to *b*.

The guru, on the other hand, may have been trying to get *a* to commit himself voluntarily to see to it that *r* by requesting him to see to it that *r* when she said to him "I would be happy if you could lead our group in that demonstration," as her act of saying "Then, it's a command" shows.[3] Before that, however, it was not clear to *a* whether she had requested or commanded.[4]

But if it was not clear whether she had requested or commanded, it means that uptake was not yet secured. As Austin points out, while perlocutionary acts involve "what we feel to be the

2 This example is an adaptation of the example used to illustrate how a promise and a command jointly generate a conflict between obligations in Yamada (2008, p. 96). The scenario is adapted from that of the original example by adding an element of uncertainty as to whether a command is given or a request is made.

3 If this is correct, we may here have an example of the so-called indirect speech act. We will say slightly more about it in Section 5.

4 It may be said here that she might have invited and not requested. But inviting is similar to requesting in that it allows for the possibility of refusal. So, for the sake of simplicity, let us assume that c has requested.

real production of real effects," illocutionary acts are acts that produce "what we regard as mere conventional consequences" (Austin 1955, p. 102).[5] Thus, he remarks,

> The illocutionary act 'takes effect' in certain ways, as distinguished from producing consequences in the sense of bringing about states of affairs in the 'normal' way, i.e. changes in the natural course of events (op. cit., p. 117).

Here we need to distinguish carefully the conventional consequences of illocutionary acts from the securing of uptake. As Sbisà points out, conventional effects are produced "thanks to the agreement on the part of the relevant participants on which act it is that has been performed" (Sbisà, 2007, p. 465). Since conventional effects of illocutionary acts are "mere conventional consequences," they can be produced by such an agreement, but exactly for this reason, their production depends on the securing of uptake.[6] But then, how should the situation brought about by the guru's act of saying "I would be happy if you could lead our group in that demonstration" be characterized when uptake has not been secured? This is one of the main questions we will examine in this paper.

Before starting the examination, however, let us have a brief look at a slightly different scenario, in which *a* has not been invited to the conference in São Paulo and does not have any other prior commitments that would prevent him from joining the demonstration. Suppose he is willing to help the guru. In such a situation, *a* may gladly and immediately say that he will lead the group when he hears *c*'s words without bothering himself about whether the guru has requested or commanded.[7] As Example 1 shows, however, it is sometimes important whether the possibility of refusal is allowed for or not.

So, let us go back to Example 1. For the sake of simplicity, let us assume what Searle calls the "[n]ormal input and output conditions" (Searle 1969, p. 57) are satisfied.[8] In a situation in which these conditions are satisfied, the uncertainty of uptake may still arise if the locution used is the kind of locution usable in performing two (or more) types of illocutionary acts having

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5 Page references to Austin 1955 refer to the pages in the second edition.

6 For the conventional effects, we will say more in Section 4

7 We will say slightly more about this example in Section 5.

8 They are meant to "cover the large and indefinite range of conditions under which any kind of serious and literal linguistic communication is possible. . . . Together, they include such things as that the speaker and hearer both know how to speak the language; both are conscious of what they are doing; they have no physical impediments to communication, such as deafness, aphasia, or laryngitis; and they are not acting in a play or telling jokes, etc." (ibid).

different illocutionary forces and the conditions for performing them such as those on the authority and so on are all satisfied.[9] In Example 1, we have just such a situation.

3. Action Models and Product Updates in DEL

As is mentioned in Section 1, action models and the method of product update are introduced by Baltag, Moss, & Solecki (1998). A textbook presentation of the logics (called Action Model Logics) based on their ideas can be found in van Ditmarsch, van der Hoek & Kooi (2007), where the logic that does not have common knowledge operators is called AM and the logic that has common knowledge operators is called AMC. We here briefly review the basic ideas of action models and the method of product update by analyzing an example from van Ditmarsch, van der Hoek & Kooi (2007). For the sake of simplicity, we will present a fragment AM^{-} of AM, which is obtained from AM by eliminating non-deterministic choice actions.[10]

Consider the following example.

Example 2 (van Ditmarsch, van der Hoek & Kooi 2007, p. 142.) Consider two stockbrokers Anne and Bill, having a little break in a Wall Street bar, sitting at a table. A messenger comes in and delivers a letter to Anne. On the envelope is written "urgently requested data on United Agents." Anne opens and reads the letter in the presence of Bill. (United Agents is doing well.) □

Suppose both Anne and Bill were not sure whether United Agents was doing well or not before the messenger came and this was common knowledge between them. Suppose also that the messenger announced that the letter was about whether United Agents was doing well or not in their presence (ibid.).

Let p be the proposition that United Agents is doing well. Since Anne has read the letter, she has learnt that p. Let p and np be Anne's act of learning that p and Anne's act of learning that $\neg p$, respectively. Let pre be a function such that $\mathsf{pre}(\mathsf{s})$ is the precondition for performing the act s. Then, $\mathsf{pre}(\mathsf{p}) = p$ and $\mathsf{pre}(\mathsf{np}) = \neg p$. Since Anne has read the letter, she knows p has happened, but Bill has only observed Anne's act of reading the letter, and so does not know

9 Here we have in mind the kind of conditions Austin classifies as (A. 1) and (A. 2) in his doctrine of the infelicities (Austin 1955, pp. 14–15).

10 On non-deterministic choice actions, see van Ditmarsch, van der Hoek & Kooi (2007), pp. 112f, 151f, 165.

whether what has really happened is p or np. But he learns that either p or np has happened and that Anne now knows whether p or not. Anne knows it, Bill knows that Anne knows it, Anne knows that Bill knows that Anne knows it, and so on.

Action models represent which actions agents can (or cannot) distinguish.

Definition 1 (Action Model, op. cit., p. 149.) Let $\mathcal{L}$ be any logical language based on a finite set of agents A and a countably infinite set P of proposition letters. An action model is a tuple $\mathsf{M} = (\mathsf{S}, \{\sim_i^* \mid i \in A\}, \mathsf{pre})$ such that S is a non-empty finite set of action points, $\sim_i^*$ is an equivalence relation over $\mathsf{S} \times \mathsf{S}$ for each agent $i \in A$, and pre is a function from the set S to the set of all the formulas of $\mathcal{L}$. A pointed action model is a pair of the form (M, s) such that M is an action model $(\mathsf{S}, \{\sim_i^* \mid i \in A\}, \mathsf{pre})$ and $\mathsf{s} \in \mathsf{S}$. □

Action points name possible acts. For any action points s, $\mathsf{t} \in \mathsf{S}$ and an agent $i \in A$, $\mathsf{s} \sim_i^* \mathsf{t}$ means that s and t are indistinguishable for i. For each action point $\mathsf{s} \in \mathsf{S}$, $\mathsf{pre}(\mathsf{s})$ is the precondition of s. An action s can be performed in a world w of a model $\mathcal{M}$ of the language $\mathcal{L}$ only if $\mathsf{pre}(\mathsf{s})$ holds in w.

Now, let us go back to Example 2. As Bill has learnt that either p or np has happened, let us consider an action model with the set of action points $\{\mathsf{p}, \mathsf{np}\}$. Let a and b be Anne and Bill, respectively. Then, Bill's uncertainty about what has happened can be represented by $\mathsf{p} \sim_b^* \mathsf{np}$. Since Anne knows p has happened, we also have $\mathsf{p} \not\sim_a^* \mathsf{np}$.

In the above discussion of Example 2, the availability of some suitable language to which p and $\neg p$ belong is assumed. The language presupposed may be the language of action model logic. For the sake of simplicity, however, we may work with a simpler language. Let $\mathcal{L}_{\mathsf{EL}}$ be the language of the standard multi-agent Epistemic Logic EL without the common knowledge operators based on a countably infinite set P of proposition letters and a finite set A of agents. Then, the language $\mathcal{L}_{\mathsf{AM}^-}$ of the fragment AM^- of the action model logic AM can be defined by extending $\mathcal{L}_{\mathsf{EL}}$.

Definition 2 (The language $\mathcal{L}_{\mathsf{EL}}$) Take a countably infinite set P of proposition letters and a finite set A of agents, with p ranging over P and i over A. The language $\mathcal{L}_{\mathsf{EL}}$ of EL based on P and A is given by

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid \mathsf{K}_i\varphi \,.$$

□

The language $\mathcal{L}_{\mathsf{AM}^-}$ of the fragment of action model logic AM^- can be obtained by adding dynamic modalities indexed by pointed action models to $\mathcal{L}_{\mathsf{EL}}$.

Definition 3 (The language $\mathcal{L}_{\mathsf{AM}^-}$, cf. op. cit., p. 149.) Take the same countably infinite set P of proposition letters and the same finite set A of agents, with p ranging over P and i over A. The language $\mathcal{L}_{\mathsf{AM}^-}$ of AM^- based on P and A is given by

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid \mathsf{K}_i\varphi \mid [\alpha]\varphi\ ,$$
$$\alpha\ ::= (\mathsf{M}, \mathsf{s})\ ,$$

where (M, s) is a pointed action model with a finite set of action points S such that $\mathsf{s} \in \mathsf{S}$, and for any $\mathsf{t} \in \mathsf{S}$, $\mathsf{pre}(\mathsf{t})$ is a formula of this language that has already been constructed in a previous stage of the inductively defined hierarchy. □

The formula of the form $[\alpha]\varphi$ means that whenever an event of the form α happens, φ holds in the resulting situation.

Note that the symbol 'p' in these definitions is a metavariable for proposition letters, but the 'p' in the preceding discussions and the subsequent discussions of Example 2 is assumed to be the proposition that United Agents is doing well.

Formulas of $\mathcal{L}_{\mathsf{AM}^-}$ are interpreted with reference to $\mathcal{L}_{\mathsf{EL}}$-models.

Definition 4 ($\mathcal{L}_{\mathsf{EL}}$-model) Given a countably infinite set P of proposition letters and a finite set A of agents, with p ranging over P and i over A, an $\mathcal{L}_{\mathsf{EL}}$-model is a tuple $\mathcal{M} = (W, \{\sim_i \mid i \in A\}, V)$ such that W is a non-empty set, $\sim_i$ is an equivalence relation over $W \times W$, and V is a function that assigns a subset $V(p)$ of W to each proposition letter $p \in \mathsf{P}$.[11] □

Definition 5 ($\vDash_{\mathsf{AM}^-}$, cf. op. cit., p. 151.) Given the language $\mathcal{L}_{\mathsf{AM}^-}$ based on a countably infinite set P of proposition letters and a finite set A of agents, with p ranging over P and i over A, let $\mathcal{M} = (W, \{\sim_i \mid i \in A\}, V)$ be an $\mathcal{L}_{\mathsf{EL}}$-model and (M, s) be a pointed action model based on $\mathcal{L}_{\mathsf{AM}^-}$. If $w \in W$, $p \in \mathsf{P}$, and φ and ψ are formulas of $\mathcal{L}_{\mathsf{AM}^-}$, then

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11 Note that $\sim_i$ is standardly required to be an equivalence relation for each $i \in A$. The validity of this requirement, however, is often disputed as it requires the agents to be idealized. On how this requirement should be modified, however, no agreement seems to have been established. We leave this issue for further study and follow the standard treatment in this paper.

(a) $\mathcal{M}, w \vDash_{\mathsf{AM}^-} p$ iff $w \in V(p)$
(b) $\mathcal{M}, w \vDash_{\mathsf{AM}^-} \neg\varphi$ iff $\mathcal{M}, w \nvDash_{\mathsf{AM}^-} \varphi$,
(c) $\mathcal{M}, w \vDash_{\mathsf{AM}^-} (\varphi \wedge \psi)$ iff $\mathcal{M}, w \vDash_{\mathsf{AM}^-} \varphi$ and $\mathcal{M}, w \vDash_{\mathsf{AM}^-} \psi$,
(d) $\mathcal{M}, w \vDash_{\mathsf{AM}^-} \mathsf{K}_i\varphi$ iff for every v such that w $\sim_i v$, $\mathcal{M}, v \vDash_{\mathsf{AM}^-} \varphi$, and
(e) $\mathcal{M}, w \vDash_{\mathsf{AM}^-} [(\mathsf{M}, \mathsf{s})]\varphi$ iff $\mathcal{M}, w \vDash_{\mathsf{AM}^-} \mathsf{pre}(\mathsf{s})$ implies $\mathcal{M} \otimes \mathsf{M}, (w, \mathsf{s}) \vDash_{\mathsf{AM}^-} \varphi$,

where $\mathcal{M} \otimes \mathsf{M}$ is a tuple $(W^{\otimes}, \{\sim_i^{\otimes} | i \in A\}, V^{\otimes})$ such that

(i) $W^{\otimes} = \{(w, \mathsf{s}) \in W \times \mathsf{S} \mid \mathcal{M}, w \vDash_{\mathsf{AM}^-} \mathsf{pre}(\mathsf{s})\}$,
(ii) $(w, \mathsf{s}) \sim_i^{\otimes} (v, \mathsf{t})$ iff $w \sim_i v$ and $\mathsf{s} \sim_i^{*} \mathsf{t}$ for any $(w, \mathsf{s}), (v, \mathsf{t}) \in W^{\otimes}$, and
(iii) $V^{\otimes}(p) = \{(w, \mathsf{s}) \in W^{\otimes} | w \in V(p)\}$. □

$\mathcal{M} \otimes \mathsf{M}$ is the result of updating $\mathcal{M}$ with M. $(w, \mathsf{s}) \sim_i^{\otimes} (v, \mathsf{t})$ means that (w, s) and (v, t) are indistinguishable for i. As $\sim_i$ and $\sim_i^{*}$ are equivalence relations, $\sim_i^{\otimes}$ is also an equivalence relation. Thus, $\mathcal{M} \otimes \mathsf{M}$ is an $\mathcal{L}_{\mathsf{EL}}$-model.

We can now define a specific pointed action model that represents the event in which Anne reads the letter but Bill only observes Anne's act of reading it.

Definition 6 (The pointed action model (Read, p), cf. op. cit., p. 142.) Given the language $\mathcal{L}_{\mathsf{AM}^-}$ based on a countably infinite set P of proposition letters such that $p \in \mathsf{P}$ and a finite set A of agents such that $A = \{a, b\}$, the pair (Read, p) is a pointed action model such that Read is a tuple

$$(\mathsf{S}, \{\sim_i^{*} | i \in A\}, \mathsf{pre}),$$

where

(i) $\mathsf{S} = \{\mathsf{p}, \mathsf{np}\}$,
(ii) $\sim_a^{*} = \{(\mathsf{p},\mathsf{p}), (\mathsf{np},\mathsf{np})\}$, $\sim_b^{*} = \{(\mathsf{p},\mathsf{p}), (\mathsf{np},\mathsf{np}), (\mathsf{p},\mathsf{np}), (\mathsf{np},\mathsf{p})\}$,
(iii) $\mathsf{pre}(\mathsf{p}) = p$, $\mathsf{pre}(\mathsf{np}) = \neg p$. □

This is a specific instance of (M, s) in Definition 3.

We then illustrate how the event (Read, p) changes cognitive states of Anne and Bill in Example 2. First, let us build an $\mathcal{L}_{\mathsf{EL}}$-model that represents the situation just before Anne read the letter.

Definition 7 (The model $\mathcal{M}_E$) Let $\mathcal{M}_E = (W, \{\sim_i | i \in A\}, V)$ be an $\mathcal{L}_{\mathsf{EL}}$-model such that $W = \{w, v\}$, $A = \{a, b\}$, $\sim_a = \sim_b = W \times W$, and $V(p) = \{w\}$. □

Since $w \sim_a w$, $w \sim_a v$, $v \sim_a v$, $v \sim_a w$, and $V(p) = \{w\}$, we have

$$\mathcal{M}_E, w \models_{\mathsf{AM}^-} \neg \mathsf{K}_a p \wedge \neg \mathsf{K}_a \neg p$$

and

$$\mathcal{M}_E, v \models_{\mathsf{AM}^-} \neg \mathsf{K}_a p \wedge \neg \mathsf{K}_a \neg p.$$

Similarly, since $w \sim_b w$, $w \sim_b v$, $v \sim_b v$, and $v \sim_b w$, we have

$$\mathcal{M}_E, w \models_{\mathsf{AM}^-} \neg \mathsf{K}_b p \wedge \neg \mathsf{K}_b \neg p$$

and

$$\mathcal{M}_E, v \models_{\mathsf{AM}^-} \neg \mathsf{K}_b p \wedge \neg \mathsf{K}_b \neg p.$$

Moreover, we have

$$\begin{gathered}
\mathcal{M}_E, w \models_{\mathsf{AM}^-} \mathsf{K}_a(\neg \mathsf{K}_b p \wedge \neg \mathsf{K}_b \neg p), \\
\mathcal{M}_E, w \models_{\mathsf{AM}^-} \mathsf{K}_b(\neg \mathsf{K}_a p \wedge \neg \mathsf{K}_a \neg p), \\
\mathcal{M}_E, v \models_{\mathsf{AM}^-} \mathsf{K}_a(\neg \mathsf{K}_b p \wedge \neg \mathsf{K}_b \neg p), \\
\mathcal{M}_E, v \models_{\mathsf{AM}^-} \mathsf{K}_b(\neg \mathsf{K}_a p \wedge \neg \mathsf{K}_a \neg p), \\
\mathcal{M}_E, w \models_{\mathsf{AM}^-} \mathsf{K}_b \mathsf{K}_a(\neg \mathsf{K}_b p \wedge \neg \mathsf{K}_b \neg p), \\
\mathcal{M}_E, w \models_{\mathsf{AM}^-} \mathsf{K}_a \mathsf{K}_b(\neg \mathsf{K}_a p \wedge \neg \mathsf{K}_a \neg p),
\end{gathered}$$

and so on. Thus, $(\mathcal{M}_E, w)$ is suitable for representing the situation just before Anne read the letter.

Next, let us examine how the product update works when applied to $(\mathcal{M}_E, w)$ and (Read, p).

Fact 1 ($\mathcal{M}_E \otimes$ Read) Since pre(p) $= p$ and pre(np) $= \neg p$, $\mathcal{M}_E \otimes$ Read is a tuple

$$(W \otimes A, \{\sim_i^{\otimes} \mid i \in A\}, V^{\otimes})$$

such that

(i) $W^{\otimes} = \{(w,\mathsf{p}),(v,\mathsf{np})\}$,
(ii) $\sim_a^{\otimes} = \{((w,\mathsf{p}),(w,\mathsf{p})),((v,\mathsf{np}),(v,\mathsf{np}))\}$,
(iii) $\sim_b^{\otimes} = \{((w,\mathsf{p}),(w,\mathsf{p})),((v,\mathsf{np}),(v,\mathsf{np})),((w,\mathsf{p}),(v,\mathsf{np})),((v,\mathsf{np}),(w,\mathsf{p}))\}$, and
(iv) $V^{\otimes}(p) = \{(w,\mathsf{p})\}$. □

Just as $(\mathcal{M}_E, w)$ represents the situation before Anne read the letter, $(\mathcal{M}_E \otimes \mathsf{Read}, (w, \mathsf{p}))$ represents the situation after Anne read the letter.

Since $V^{\otimes}(p) = \{(w,\mathsf{p})\}$, $(w,\mathsf{p}) \sim_a^{\otimes} (w,\mathsf{p})$, and $(w,\mathsf{p}) \not\sim_a^{\otimes} (v,\mathsf{np})$, we have

$$\mathcal{M}_E \otimes \mathsf{Read}, (w,\mathsf{p}) \vDash_{\mathsf{AM}^-} \mathsf{K}_a p$$

This means that we have

$$\mathcal{M}_E, w \vDash_{\mathsf{AM}^-} [(\mathsf{Read},\mathsf{p})]\mathsf{K}_a p.$$

Since $(w,\mathsf{p}) \sim_b^{\otimes} (v,\mathsf{np})$, however, we have

$$\mathcal{M}_E \otimes \mathsf{Read}, (w,\mathsf{p}) \vDash_{\mathsf{AM}^-} \neg\mathsf{K}_b p \wedge \neg\mathsf{K}_b \neg p.$$

This means that we have

$$\mathcal{M}_E, w \vDash_{\mathsf{AM}^-} [(\mathsf{Read},\mathsf{p})](\neg\mathsf{K}_b p \wedge \neg\mathsf{K}_b \neg p).$$

Moreover, it is not difficult to show that we have

$$\mathcal{M}_E, w \vDash_{\mathsf{AM}^-} [(\mathsf{Read},\mathsf{p})]\mathsf{K}_a(\neg\mathsf{K}_b p \wedge \neg\mathsf{K}_b \neg p),$$
$$\mathcal{M}_E, v \vDash_{\mathsf{AM}^-} [(\mathsf{Read},\mathsf{np})]\mathsf{K}_a(\neg\mathsf{K}_b p \wedge \neg\mathsf{K}_b \neg p),$$
$$\mathcal{M}_E, w \vDash_{\mathsf{AM}^-} [(\mathsf{Read},\mathsf{p})]\mathsf{K}_b\mathsf{K}_a(\neg\mathsf{K}_b p \wedge \neg\mathsf{K}_b \neg p),$$
$$\mathcal{M}_E, v \vDash_{\mathsf{AM}^-} [(\mathsf{Read},\mathsf{np})]\mathsf{K}_a \neg p,$$
$$\mathcal{M}_E, w \vDash_{\mathsf{AM}^-} [(\mathsf{Read},\mathsf{p})]\mathsf{K}_b(\mathsf{K}_a p \vee \mathsf{K}_a \neg p),$$

$$\mathcal{M}_E, w \vDash_{\mathsf{AM}^-} [(\mathsf{Read},\mathsf{p})]\mathsf{K}_a\mathsf{K}_b(\mathsf{K}_a p \lor \mathsf{K}_a\neg p)\,,$$

and so on. These results show that AM^- nicely reflects the difference between the effects of the event represented by $(\mathsf{Read}, \mathsf{p})$ on Anne and Bill. In $(\mathcal{M}_E \otimes \mathsf{Read}, (w, \mathsf{p}))$, for example, Anne knows that p holds, but Bill does not know whether p holds or not. Bill, however, knows that Anne knows whether p holds or not and Anne knows that Bill knows it.[12]

4. When Uptake Is Secured

Let us move on to acts of commanding and requesting. The following is yet another variant of Example 1.

Example 3 Suppose an agent a has an on-line meeting with c and d. The agent c is the guru of the political group of which a and d are members. She (the guru) says, "There will be an important demonstration in Tokyo on August 1st next year." Then she says to a, "I would be happy if you would lead our group in that demonstration." Upon hearing it, a correctly understands that she has requested him to lead the group in the demo. The agent d also hears her words, but he starts wondering whether it is a request or a command. □

Is it possible to apply the method of product update to such a scenario?

In order to do so, we need to take the fact that illocutionary acts of commanding and requesting affect the deontic aspects of the situation into consideration. This means that we need to decide how to update deontic accessibility relations. Dynamified Multi-agent Epistemic Deontic Logic DMEDL defined in Yamada (2016) is useful here as it is designed to deal with acts of commanding and requesting. Just as DEL is developed by adding dynamic modalities to static epistemic logic EL, DMEDL is developed by adding dynamic modalities to static deontic logic MEDL. We now review the language and its semantics defined in Yamada (2016). First, let us have a look at the language of the static base logic.[13]

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12 For the proofs of the completeness of AM and AMC, see van Ditmarsch, van der Hoek & Kooi (2007, pp. 194–201).

13 Yamada (2016) includes $\top$ as a primitive, but we do not include it as it can be introduced as an abbreviation of $(q \to q)$ for some fixed proposition letter q.

Definition 8 (The language $\mathcal{L}_{\mathsf{MEDL}}$ (Yamada 2016, p. 483.)) Take a countably infinite set P of proposition letters and a finite set A of agents, with p ranging over P and i, j, k over A. The language $\mathcal{L}_{\mathsf{MEDL}}$ of the multi-agent epistemic deontic logic MEDL is given by

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid \mathsf{K}_i\varphi \mid \mathsf{O}_{(i,j,k)}\varphi\ .$$

□

This language is extended to the language of dynamified logic DMEDL.

Definition 9 (The language $\mathcal{L}_{\mathsf{DMEDL}}$, op. cit., p. 484.) Take the same countably infinite set P of proposition letters and the same finite set A of agents as above, with p ranging over P and i, j, k over A. The language $\mathcal{L}_{\mathsf{DMEDL}}$ of dynamified multi-agent epistemic deontic logic DMEDL is given by

$$\begin{aligned} \varphi &::= p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid \mathsf{K}_i\varphi \mid \mathsf{O}_{(i,j,k)}\varphi \mid [\pi]\varphi\ , \\ \pi &::= \mathrm{Command}_{(i,j)}\varphi \mid \mathrm{Promise}_{(i,j)}\varphi \mid \mathrm{Request}_{(i,j)}\varphi\ . \end{aligned}$$

Operators of the form $[\mathrm{Command}_{(i,j)}\varphi]$, $[\mathrm{Promise}_{(i,j)}\varphi]$, and $[\mathrm{Request}_{(i,j)}\varphi]$ are called command operators, promise operators, and request operators, respectively. They are also called action operators. □

The formula of the form $\mathsf{O}_{(i,j,k)}\varphi$ intuitively means that it is obligatory for the agent i to see to it that φ towards the agent j due to the agent k, where i is the agent who owes the obligation to see to it that φ, j is the agent to whom this obligation is owed, and k is the agent who creates this obligation. The three roles of agents involved are distinguished in order to differentiate the obligation generated by acts of commanding, promising, and requesting, as we shall see below.

The expression of the form $\mathrm{Command}_{(i,j)}\varphi$ stands for the type of acts in which the agent i (the command issuer) commands the agent j (the commandee) to see to it that φ, the expression of the form $\mathrm{Promise}_{(i,j)}\varphi$ stands for the type of acts in which the agent i (the promiser) promises the agent j (the promisee) that she will see to it that φ, and the expression of the form $\mathrm{Request}_{(i,j)}\varphi$ stands for the type of acts in which the agent i (the requester) requests the agent j (the requestee) to see to it that φ. The formula of the form $[\pi]\varphi$ means that whenever an act of the form π is performed, φ holds in the resulting situation.

The formulas of $\mathcal{L}_{\mathsf{MEDL}}$ and $\mathcal{L}_{\mathsf{DMEDL}}$ are interpreted with reference to $\mathcal{L}_{\mathsf{MEDL}}$models.

Definition 10 ($\mathcal{L}_{\mathsf{MEDL}}$-model, op, cit., p. 483.) Given a countably infinite set P of proposition letters and a finite set A of agents, an $\mathcal{L}_{\mathsf{MEDL}}$-model is a tuple

$$\mathcal{M} = (W^{\mathcal{M}}, \{\sim_i^{\mathcal{M}} \mid i \in A\}, \{D_{(i,j,k)}^{\mathcal{M}} \mid i,j,k \in A\}, V^{\mathcal{M}})$$

such that

(i) $W^{\mathcal{M}}$ is a non-empty set (heuristically, of 'possible worlds'),
(ii) $\sim_i^{\mathcal{M}}$ is an equivalence relation over $W^{\mathcal{M}} \times W^{\mathcal{M}}$,
(iii) $D_{(i,j,k)}^{\mathcal{M}}$ is a subset of $W^{\mathcal{M}} \times W^{\mathcal{M}}$, and
(iv) $V^{\mathcal{M}}$ is a function that assigns a subset $V^{\mathcal{M}}(p)$ of $W^{\mathcal{M}}$ to each proposition letter $p \in \mathsf{P}$. □

Note that deontic accessibility relations ($D_{(i,j,k)}^{\mathcal{M}}$)'s are not required to be serial. As they are used to interpret deontic operators in the semantics defined below, supposing them to be serial precludes the possibility of conflicts of obligations. As DMEDL deals with examples in which a pair of commands or a pair of a command and a promise, for example, generate conflicting obligations, deontic accessibility relations are not required to be serial (Yamada 2011, pp. 64, 68–70).

Before going into the truth definition, a brief comparison of the acts of commanding and requesting is in order here. As we have seen, acts of commanding do not allow for the option of refusal. So, when you are commanded to do something, it becomes obligatory for you to do what you are commanded to do. In the case of acts of requesting, however, the option of refusal is allowed for. Thus, even if you are requested to do something, for example, it does not make it obligatory for you to do what you are requested to do. It would be blameworthy, however, if you ignore the request completely and behave as if no request is made. You need at least to respond. A positive response in this context means that you agree to do what is requested and a negative response means that you refuse. Thus, it becomes obligatory for you to decide whether you should do it or not and let the requester know your decision.

The following truth definition for DMEDL reflects this contrast.

Definition 11 (Op. cit., pp. 484–485.) Given the language $\mathcal{L}_{\mathsf{DMEDL}}$ based on a countably infinite set P of proposition letters and a finite set A of agents, let $\mathcal{M}$ be an $\mathcal{L}_{\mathsf{MEDL}}$-model and w a world in $W^{\mathcal{M}}$. If $p \in \mathsf{P}$, φ and ψ are formulas of $\mathcal{L}_{\mathsf{DMEDL}}$, and $i, j, k \in A$, then the relation $\vDash_{\mathsf{DMEDL}}$ is defined as follows.

(a) $\mathcal{M}, w \vDash_{\mathsf{DMEDL}} p$ iff $w \in V^{\mathcal{M}}(p)$,
(b) $\mathcal{M}, w \vDash_{\mathsf{DMEDL}} \neg\varphi$ iff $\mathcal{M}, w \nvDash_{\mathsf{DMEDL}} \varphi$,
(c) $\mathcal{M}, w \vDash_{\mathsf{DMEDL}} (\varphi \wedge \psi)$ iff $\mathcal{M}, w \vDash_{\mathsf{DMEDL}} \varphi$ and $\mathcal{M}, w \vDash_{\mathsf{DMEDL}} \psi$,
(d) $\mathcal{M}, w \vDash_{\mathsf{DMEDL}} \mathsf{K}_i\varphi$ iff for every v such that $(w, v) \in \sim_i^{\mathcal{M}}$, $\mathcal{M}, v \vDash_{\mathsf{DMEDL}} \varphi$,
(e) $\mathcal{M}, w \vDash_{\mathsf{DMEDL}} \mathsf{O}_{(i,j,k)}\varphi$ iff for every v such that $(w, v) \in D^{\mathcal{M}}_{(i,j,k)}$, $\mathcal{M}, v \vDash_{\mathsf{DMEDL}} \varphi$,
(f) $\mathcal{M}, w \vDash_{\mathsf{DMEDL}} [\mathrm{Command}_{(i,j)}\psi]\varphi$ iff $\mathcal{M}_{\mathrm{Command}_{(i,j)}\psi}, w \vDash_{\mathsf{DMEDL}} \varphi$,
(g) $\mathcal{M}, w \vDash_{\mathsf{DMEDL}} [\mathrm{Promise}_{(i,j)}\psi]\varphi$ iff $\mathcal{M}_{\mathrm{Promise}_{(i,j)}\psi}, w \vDash_{\mathsf{DMEDL}} \varphi$, and
(h) $\mathcal{M}, w \vDash_{\mathsf{DMEDL}} [\mathrm{Request}_{(i,j)}\psi]\varphi$ iff $\mathcal{M}_{\mathrm{Request}_{(i,j)}\psi}, w \vDash_{\mathsf{DMEDL}} \varphi$,

where

(i) $\mathcal{M}_{\mathrm{Command}(i,j)\psi}$ is the $\mathcal{L}_{\mathsf{MEDL}}$-model obtained from $\mathcal{M}$ by replacing $D^{\mathcal{M}}(j, i, i)$ with $\{(s, t) \in D^{\mathcal{M}}(j, i, i) \mid \mathcal{M}, t \vDash_{\mathsf{DMEDL}} \psi\}$,
(ii) $\mathcal{M}_{\mathrm{Promise}_{(i,j)}\psi}$ is the $\mathcal{L}_{\mathsf{MEDL}}$-model obtained from $\mathcal{M}$ by replacing $D^{\mathcal{M}}(i, j, i)$ with $\{(s, t) \in D^{\mathcal{M}}(i, j, i) \mid \mathcal{M}, t \vDash_{\mathsf{DMEDL}} \psi\}$, and
(iii) $\mathcal{M}_{\mathrm{Request}_{(i,j)}\psi}$ is the $\mathcal{L}_{\mathsf{MEDL}}$-model obtained from $\mathcal{M}$ by replacing $D^{\mathcal{M}}(j, i, i)$ with $\{(s, t) \in D^{\mathcal{M}}(j, i, i) \mid \mathcal{M}, t \vDash_{\mathsf{DMEDL}} (\mathsf{K}_i\mathsf{O}_{(j.i.j)}\psi \vee \mathsf{K}_i\neg\mathsf{O}_{(j.i.j)}\psi)\}$. □

This definition supports the following principles.

Proposition 1 (Op. cit., p. 485.) The following principles hold.

(CUGO) If φ is free of occurrences of $\mathsf{O}_{(j,\, i,\, i)}$, $[\mathrm{Command}_{(i,j)}\varphi]\mathsf{O}_{(j,\, i,\, i)}\varphi$ is valid.
(PUGO) If φ is free of occurrences of $\mathsf{O}_{(i,\, j,\, i)}$, $[\mathrm{Promise}_{(i,j)}\varphi]\mathsf{O}_{(i,j,\, i)}\varphi$ is valid.
(RUGO) If φ is free of occurrences of $\mathsf{O}_{(j,\, i,\, i)}$,

$$[\mathrm{Request}_{(i,j)}\varphi]\mathsf{O}_{(j,\, i,\, i)}(\mathsf{K}_i\mathsf{O}_{(j,\, i,j)}\varphi \vee \mathsf{K}_i\neg\mathsf{O}_{(j,\, i,j)}\varphi) \text{ is valid.}$$ □

These principles state that acts of commanding, promising, and requesting usually generate obligations.[14] In the case of acts of commanding, the commandee owes the obligation to do

14 On the restriction of the occurrences of $\mathsf{O}_{(j,\, i,\, i)}$ in φ in (CUGO) and (RUGO) and those of $\mathsf{O}_{(i,\, j,\, i)}$ in φ in (PUGO), see the discussion of a similar principle in Yamada (2007, p. 9).

what is commanded and this obligation is created by the command issuer, but in the case of acts of promising, the agent who owes the obligation is identical with the agent who creates the obligation, and thus, the obligation generated by an act of promising represents to what course of action the promiser has committed herself.

Then, compare an act of requesting with an act of commanding. In the case of an act of requesting, the requestee does not owe the obligation to do what is requested (i.e,, seeing to it that φ); the option of acceptance or refusal is allowed for. As we have stated, however, the requestee owes the obligation to make a response. This obligation is created by the requester, but it only means that the requestee needs to decide whether she should do what is requested or not and let the requester know her decision. Here the positive answer amounts to see to it that $\mathsf{K}_i\mathsf{O}_{(j.i.j)}\varphi$ (i.e., to see to it that the requester knows that the requestee commits herself to seeing to it that φ), while the negative answer amounts to seeing to it that $\mathsf{K}_i\neg\mathsf{O}_{(j,\ i,j)}\varphi$) (i.e., letting the requester knows that she does not commit herself to seeing to it that φ).

When what is requested (i.e., to see to it that φ) can be done on the spot, the requestee might do it without saying anything. (RUGO) does not count this as the third option distinct from the positive and the negative responses. We may understand such a response as skipping to the sequel of the implicit positive response. Thus, Definition 11 incorporates a version of "the two-option analysis of requesting."[15]

In addition to the above three principles, DMEDL validates the following principles.[16]

Proposition 2 The following principles hold.

(CUGU) If φ is free of occurrences of $\mathsf{O}_{(j,i,i)}$, $[\text{Command}_{(i,j)}\varphi]\mathsf{K}_j\mathsf{O}_{(j,i,i)}\varphi$ is valid.
(PUGU) If φ is free of occurrences of $\mathsf{O}_{(i,j,i)}$, $[\text{Promise}_{(i,j)}\varphi]\mathsf{K}_j\mathsf{O}_{(i,j,i)}\varphi$ is valid.
(RUGU) If φ is free of occurrences of $\mathsf{O}_{(j,i,i)}$,

$$[\text{Request}_{(i,j)}\varphi]\mathsf{K}_j\mathsf{O}_{(j,i,i)}(\mathsf{K}_i\mathsf{O}_{(j.i.j)}\varphi \vee \mathsf{K}_i\neg\mathsf{O}_{(j.i.j)}\varphi) \text{ is valid.} \quad \square$$

15 Traum and Allen (1994, pp. 3, 5) also give a similar analysis. They include only the options of accepting and refusing in their discussion of obligations as effects of acts of requesting. We will show in Appendix that the deontic action model logic based on the three-option analysis of acts of requesting fails to capture *d*'s uncertainty in Example 3. This will give a clear reason for preferring the two-option analysis over the three-option analysis.

Regarding the completeness of DMEDL, a proof system and the outline of the proof of the completeness of the system based on the three-option analysis of requesting are presented in Yamada (2011, pp. 75–77). A proof system of DMEDL based on the two-option analysis and a proof of its completeness can be obtained from the proof system based on the three-option analysis and the proof of its completeness, respectively, *mutatis mutandis*.

16 The three-option analogues of these principles are stated in Yamada (2011, pp. 73, 75).

These principles state that acts of commanding, promising, and requesting usually secure uptake. This is favorable as the securing of uptake is necessary for performing illocutionary acts.

The semantics of DMEDL, however, also validates the following stronger principles.

Proposition 3 The following principles hold.

(SCUGU) If φ is free of occurrences of $\mathsf{O}_{(j,i,i)}$, $[\text{Command}_{(i,j)}\varphi]\text{K}_k\mathsf{O}_{(j,i,i)}\varphi$ is valid.
(SPUGU) If φ is free of occurrences of $\mathsf{O}_{(i,j,i)}$, $[\text{Promise}_{(i,j)}\varphi]\text{K}_k\mathsf{O}_{(i,j,i)}\varphi$ is valid.
(SRUGU) If φ is free of occurrences of $\mathsf{O}_{(j,i,i)}$,

$$[\text{Request}_{(i,j)}\varphi]\text{K}_k\mathsf{O}_{(j,i,i)}(\mathsf{K}_i\mathsf{O}_{(j.i.j)}\varphi \vee \mathsf{K}_i\neg\mathsf{O}_{(j.i.j)}\varphi) \text{ is valid.} \qquad \Box$$

As the model updating operation that interprets an act of commanding of the form $\text{Command}_{(i,j)}\varphi$, for example, cuts every deontic accessibility link (arrow) for (j, i, i) that arrives in a world in which φ does not hold before the update according to Clause (i) in Definition 11, any world in which φ does not hold before the update will be inaccessible through the deontic accessibility links for (j, i, i) after the update. Thus, if φ is free of occurrences of $\mathsf{O}_{(j,i,i)}$, $\mathsf{O}_{(j,i,i)}\varphi$ holds in every world after the update, and so, it holds in every world epistemically accessible for any agent k after the update. This excludes the possibility of uncertainty of uptake and makes it difficult to model the scenario in Example 3 in DMEDL. Thus, our task here is to avoid this difficulty by defining a deontic version of product update.

For this purpose, we need to extend the language of DMEDL. DMEDL is designed to characterize how successfully performed acts of commanding, promising, and requesting change situations, and it only deals with the situations in which their preconditions are assumed to be satisfied, but the definition of the product update requires us to include the precondition function pre, as we have seen in Section 3. For the sake of simplicity, let us ignore acts of promising. In the case of acts of requesting, we may assume their preconditions to be a tautology $\top$ defined as an abbreviation for $\neg(q \wedge \neg q)$ for some fixed proposition letter $q \in \mathsf{P}$. For the case of acts of commanding, let us add a set of formulas of the form $\mathsf{Auth}_{(i,j,h)}\varphi$, meaning that an agent i is authorized by an organization h to command an agent j to see to it that φ, and a finite set H of authorizing organizations.

Definition 12 (The language $\mathcal{L}_{\text{EMEDL}}$) Take a countably infinite set P of proposition letters, a finite set A of agents, and a finite set H of organizations, with p ranging over P, i, j, k ranging over A, and h ranging over H. The language $\mathcal{L}_{\mathsf{EMEDL}}$ of the extended multi-agent epistemic deontic logic EMEDL is given by

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid \mathsf{K}_i\varphi \mid \mathsf{O}_{(i,j,k)}\varphi \mid \mathsf{Auth}_{(i,j,h)}\varphi.$$

The set of all well-formed formulas (sentences) of $\mathcal{L}_{\mathsf{EMEDL}}$ is denoted by S_{EMEDL}. Operators of the form $\mathsf{O}_{(i,j,k)}$ and operators of the form $\mathsf{Auth}(i, j, h)$ are called deontic operators. For each $i, j, k \in A$, we call a sentence $\mathsf{O}_{(i,j,k)}$-free if the operator $\mathrm{O}_{(i,j,k)}$ does not occur in it. For each $i, j \in A$ and $h \in H$, we call a sentence $\mathsf{Auth}(i, j, h)$-free if the operator of the form $\mathsf{Auth}(i, j, h)$ does not occur in it. We call a sentence epistemic if no deontic operators occur in it, and Boolean if no modal operators occur in it. For each i, j, $k \in A$, the set of all $\mathsf{O}(i, j,k)$-free sentences is denoted by $S_{\mathsf{O}_{(i,j,k)}\text{-free}}$, and for each $i, j \in A$ and $h \in H$, the set of $\mathsf{Auth}(i, j, h)$-free sentences are denoted by $S_{\mathsf{Auth}_{(i,j,h)}\text{-free}}$. The set of all epistemic sentences and the set of all Boolean sentences are denoted by $S_{\mathrm{Epistemic}}$ and S_{Boole}, respectively. □

The formulas of $\mathcal{L}_{\mathsf{EMEDL}}$ are interpreted with reference to $\mathcal{L}_{\mathsf{EMEDL}}$-models.

Definition 13 ($\mathcal{L}_{\mathsf{EMEDL}}$-model) Given the language $\mathcal{L}_{\mathsf{EMEDL}}$ based on a countably infinite set P of proposition letters, a finite set A of agents, and a finite set H of organizations, an $\mathcal{L}_{\mathsf{EMEDL}}$-model is a tuple

$$\mathcal{M} = (W, E, D, O, V)$$

such that

(i) W is a non-empty set (heuristically, of ‘possible worlds’),
(ii) $E = \{ \sim_i^{\mathcal{M}} \mid i \in A\}$ such that $\sim_i^{\mathcal{M}}$ is an equivalence relation over $W \times W$,
(iii) $D = \{ D_{(i,j,k)}^{\mathcal{M}} \mid i, j, k \in A\}$ such that $D_{(i,j,k)}^{\mathcal{M}} \subseteq (W \times W)$,
(iv) O is a function that assigns a subset $O(i, j, h, \varphi)$ of W to each
    $(i, j, h, \varphi) \in A \times A \times H \times S_{\mathsf{EMEDL}}$, and
(v) V is a function that assigns a subset $V(p)$ of W to each proposition letter $p \in \mathsf{P}$. □

Note that the epistemic accessibility relations of the form $\sim_i^{\mathcal{M}}$ are required to be equivalence relations following the standard treatment.

We then define the satisfaction relation for the formulas of $\mathcal{L}_{\mathsf{EMEDL}}$.

Definition 14 ($\vDash_{\mathsf{EMEDL}}$) Given the language $\mathcal{L}_{\mathsf{EMEDL}}$ based on a countably infinite set P of proposition letters, a finite set A of agents, and a finite set H of organizations, let $\mathcal{M}$ be an $\mathcal{L}_{\mathsf{EMEDL}}$-model and w a point in $\mathcal{M}$. If $p \in \mathsf{P}$, $\varphi, \psi \in S_{\mathsf{EMEDL}}$, $i, j, k \in A$, and $h \in H$, then the satisfaction relation $\vDash_{\mathsf{EMEDL}}$ is defined as follows.

(a) $\mathcal{M}, w \vDash_{\mathsf{EMEDL}} p$ iff $w \in V(p)$,
(b) $\mathcal{M}, w \vDash_{\mathsf{EMEDL}} \neg\varphi$ iff $\mathcal{M}, w \nvDash_{\mathsf{EMEDL}} \varphi$,
(c) $\mathcal{M}, w \vDash_{\mathsf{EMEDL}} (\varphi \wedge \psi)$ iff $\mathcal{M}, w \vDash_{\mathsf{EMEDL}} \varphi$ and $\mathcal{M}, w \vDash_{\mathsf{EMEDL}} \psi$,
(d) $\mathcal{M}, w \vDash_{\mathsf{EMEDL}} \mathsf{K}_i\varphi$ iff for every v such that $(w, v) \in \sim_i^{\mathcal{M}}$, $\mathcal{M}, v \vDash_{\mathsf{EMEDL}} \varphi$,
(e) $\mathcal{M}, w \vDash_{\mathsf{EMEDL}} \mathsf{O}_{(i,j,k)}\varphi$ iff for every v such that $(w, v) \in D^{\mathcal{M}}_{(i,j,k)}$, $\mathcal{M}, v \vDash_{\mathsf{EMEDL}} \varphi$, and
(f) $\mathcal{M}, w \vDash_{\mathsf{EMEDL}} \mathsf{Auth}(i, j, h)\varphi$ iff $w \in O(i, j, h, \varphi)$. □

Note that Definition 13 imposes no systematic conditions on the relation between $O(i, j, h, \varphi)$ and $O(i, j, h, (\varphi \wedge \psi))$, for example. If we wish to have some logic relation between $\mathsf{Auth}(i, j, h)\varphi$ and $\mathsf{Auth}(i, j, h)(\varphi \wedge \psi)$, we need to impose some constraints on the relation between $O(i, j, h, \varphi)$ and $O(i, j, h, (\varphi \wedge \psi))$. We will not pursue this possibility here, however, as what constraints to impose seems to be a matter of choice of each organization at least to some extent. For example, suppose we have $\mathsf{Auth}(i, j, h)\varphi$ and $\mathsf{Auth}(i, j, h)\psi$. This does not seem to imply that we have $\mathsf{Auth}(i, j, h)(\varphi \wedge \psi)$. There may be an organization that finds it safe to authorize some of its members to command some other members to see to it that φ or to see to it that ψ, but finds it dangerous to authorize them to do both for some φ and ψ. Moreover, there are cases where it is reasonable for h to authorize i to command j to see to it that $\neg p$ if it authorizes i to command j to see to it that p, but it might not be reasonable to do so for some p. For example, consider the case where p means that whistle-blowers are protected.

Another thing worth noting here about Definition 13 is the following. Different kinds of illocutionary acts have different kinds of preconditions, and they update models in different ways. Thus, $\mathcal{L}_{\mathsf{EMEDL}}$ is not a universal static base language for dealing with illocutionary acts in general. It is just designed to deal with acts of commanding and requesting. Similar remarks apply to deontic action models and the languages of deontic action model logics, as we will see soon.

We now define the language $\mathcal{L}_{\mathsf{DAM}^-}$ of the deontic action model logic DAM^- by adding dynamic modalities indexed by deontic action models to $\mathcal{L}_{\mathsf{EMEDL}}$.

Definition 15 (The language $\mathcal{L}_{\mathsf{DAM}^-}$) Take the same countably infinite set P of proposition letters, the same finite set A of agents, and the same finite set H of organizations as above, with p ranging over P, i, j, k ranging over A, and h ranging over H. The language $\mathcal{L}_{\mathsf{DAM}^-}$ of deontic action model logic DAM^- is given by

$$\begin{aligned} \varphi &::= p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid \mathsf{K}_i\varphi \mid \mathsf{O}_{(i,j,k)}\varphi \mid \mathsf{Auth}_{(i,j,h)}\varphi \mid [\alpha]\varphi, \\ \alpha &::= (\mathsf{M}, \mathsf{s}), \end{aligned}$$

where (M, s) is a pointed deontic action model with a finite set of action points S such that $\mathsf{s} \in \mathsf{S}$, and for any $\mathsf{t} \in \mathsf{S}$, $\mathsf{pre}(\mathsf{t})$ is a formula of this language that has already been constructed in a previous stage of the inductively defined hierarchy. The set of all well-formed formulas of $\mathcal{L}_{\mathsf{DAM}^-}$ is denoted by S_{DAM^-}. □

Note that S_{EMEDL} is a subset of S_{DAM^-}.

We next define a specific deontic action model in order to analyze Example 3. As far as this example is concerned, we can assume that only one organization, say g, is relevant to c's authority over a. It is the political group of which a and d are members and c is the guru of g. Then, $\mathsf{Auth}_{(c,a,g)}r$ intuitively means that the organization g authorizes c to command a to see to it that r.

Since d wonders whether a command is issued or a request is made while a understands that a request is made, let com and req be an act in which c commands a to see to it that r and an act in which c requests a to see to it that r, respectively. Then, we can define a specific action model that includes com and req.

Definition 16 (The action model M^R) Given the language $\mathcal{L}_{\mathsf{DAM}^-}$ based on a countably infinite set P of proposition letters such that $r \in \mathsf{P}$, a finite set of agents $A = \{a, c, d\}$, and a finite set of organizations H such that g $\in H$, with the abbreviation $\top$ of $\neg(q \wedge \neg q)$ for some fixed proposition letter $q \in \mathsf{P}$, the action model M^R is a tuple

$$\mathsf{M}^R = (\mathsf{S}^R, \{\sim_i^R \mid i \in A\}, \mathsf{pre}^R)$$

such that

(i) $\mathsf{S}^R = \{\mathsf{com}, \mathsf{req}\}$,

(ii) $\sim_a^R = \sim_c^R = \{(\mathsf{com}, \mathsf{com}), (\mathsf{req}, \mathsf{req})\}$ and $\sim_d^R = \mathsf{S}^R \times \mathsf{S}^R$, and
(iii) $\mathsf{pre}^R(\mathsf{req}) = \top$ and $\mathsf{pre}^R(\mathsf{com}) = \mathsf{Auth}_{(c,a,g)}r$. □

Note that req and com are c's act of requesting of type Request$_{(c,a)}r$ and c's act of commanding of type Command$_{(c,a)}r$, respectively, in $\mathcal{L}_{\mathsf{DMEDL}}$, but the expressions '$\mathsf{req}$' and '$\mathsf{com}$' merely name these acts.

Also note that $\sim_i^R$ is required to be an equivalence relation. $(\mathsf{com}, \mathsf{req}) \in \sim_i^R$ means that i is not able to tell whether com has occurred or req has occurred. Thus, this model represents an event about which a and c can tell whether req has occurred or com has occurred but d cannot do so. Therefore, the pointed action model $(\mathsf{M}^R, \mathsf{req})$ is suitable for representing the event in Example 3.

Since $(\mathsf{M}^R, \mathsf{req})$ is a pointed deontic action model, the satisfaction relation $\vDash_{\mathsf{DAM}^-}$ for an instance of the language $\mathcal{L}_{\mathsf{DAM}^-}$ such that $\mathsf{M} = \mathsf{M}^R$ can be defined.

Definition 17 ($\vDash_{\mathsf{DAM}^-}$) Given the language $\mathcal{L}_{\mathsf{DAM}^-}$ based on the same countably infinite set of proposition letters P with $r \in \mathsf{P}$, the same finite set of agents $A = \{a, c, d\}$, the same finite set of organizations H with $g \in H$, and the deontic action model M^R, let $\mathcal{M}$ be an $\mathcal{L}_{\mathsf{EMEDL}}$-model (W, E, D, O, V) such that $E = \{\sim_i^{\mathcal{M}} \mid i \in A\}$ is a set of equivalence relations over $W \times W$, $D = \{D_{(i,j,k)}^{\mathcal{M}} \mid i, j, k \in A\}$, and $O(c, a, \mathrm{g}, r) = W$. If $w \in W$, $\mathsf{s} \in \mathsf{S}^R$, $p \in \mathsf{P}$, $\varphi, \psi \in S_{\mathsf{DAM}^-}$, $i, j, k \in A$, and $h \in H$, then the satisfaction relation $\vDash_{\mathsf{DAM}^-}$ is defined as follows.

(a) $\mathcal{M}, w \vDash_{\mathsf{DAM}^-} p$ iff $w \in V(p)$,
(b) $\mathcal{M}, w \vDash_{\mathsf{DAM}^-} \neg\varphi$ iff $\mathcal{M}, w \nvDash_{\mathsf{DAM}^-} \varphi$,
(c) $\mathcal{M}, w \vDash_{\mathsf{DAM}^-} (\varphi \wedge \psi)$ iff $\mathcal{M}, w \vDash_{\mathsf{DAM}^-} \varphi$ and $\mathcal{M}, w \vDash_{\mathsf{DAM}^-} \psi$,
(d) $\mathcal{M}, w \vDash_{\mathsf{DAM}^-} \mathsf{K}_i\varphi$ iff for every v such that $(w, v) \in \sim_i^{\mathcal{M}}$, $\mathcal{M}, v \vDash_{\mathsf{DAM}^-} \varphi$,
(e) $\mathcal{M}, w \vDash_{\mathsf{DAM}^-} \mathsf{O}_{(i,j,k)}\varphi$ iff for every v such that $(w, v) \in D_{(i,j,k)}^{\mathcal{M}}$, $\mathcal{M}, v \vDash_{\mathsf{DAM}^-} \varphi$,
(f) $\mathcal{M}, w \vDash_{\mathsf{DAM}^-} \mathsf{Auth}_{(i,j,h)}\varphi$ iff $\varphi \in S_{\mathsf{EMEDL}}$ and $w \in O(i, j, h, \varphi)$, and
(g) $\mathcal{M}, w \vDash_{\mathsf{DAM}^-} [(\mathsf{M}^R, \mathsf{s})]\varphi$ iff $\mathcal{M}, w \vDash_{\mathsf{DAM}^-} \mathsf{pre}^R(\mathsf{s})$ implies $\mathcal{M} \otimes \mathsf{M}^R, (w, \mathsf{s}) \vDash_{\mathsf{DAM}^-} \varphi$,

where $\mathcal{M} \otimes \mathsf{M}^R$ is a tuple

$$(W^{\otimes}, E^{\otimes}, D^{\otimes}, O^{\otimes}, V^{\otimes})$$

satisfying the following conditions.

(i) $W^{\otimes} = \{(w, \mathsf{s}) \in W \times \mathsf{S}^R \mid \mathcal{M}, w \vDash_{\mathsf{DAM}^-} \mathsf{pre}^R(\mathsf{s})\}$,

(ii) $E^{\otimes} = \{\sim^{\otimes}_i \mid i \in A\}$ such that

$(w, \mathsf{s}) \sim^{\otimes} (v, \mathsf{t})$ iff $w \sim^{\mathcal{M}}_i v$ and $\mathsf{s} \sim^{R}_i \mathsf{t}$ for any $(w, \mathsf{s}), (v, \mathsf{t}) \in W^{\otimes}$,

(iii) $D^{\otimes} = \{ D^{\otimes}_{(i,j,k)} \mid , i, j, k \in A\}$ such that

$D^{\otimes}_{(i,j,k)} = \{((w,\mathsf{s}),(v,\mathsf{t})) \in W^{\otimes} \times W^{\otimes} \mid (w,v) \in D^{\mathsf{s}}_{(i,j,k)}$ and $\mathsf{s} = \mathsf{t}\}$,

where $D^{\mathsf{s}}_{(i,j,k)} =$

(a) $D^{\mathcal{M}}_{(i,j,k)}$ if $(i,j,k) \neq (a,c,c)$,

(b) $\{(w,v) \in D^{\mathcal{M}}_{(a,c,c)} \mid \mathcal{M}, v \vDash_{\mathsf{DAM}^-} r\}$ if $\mathsf{s} = \mathsf{com}$, and

(c) $\{(w,v) \in D^{\mathcal{M}}_{(a,c,c)} \mid \mathcal{M}, v \vDash_{\mathsf{DAM}^-} (\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)\}$ if $\mathsf{s} = \mathsf{req}$,

(iv) $O^{\otimes}$ is a function that assigns a subset $O^{\otimes}(i, j, h, \varphi)$ of $W^{\otimes}$

to each $(i, j, h, \varphi) \in A \times A \times H \times S_{\mathsf{DAM}^-}$ such that $O^{\otimes}(i, j, h, \varphi) =$

(a) $(O(i, j, h, \varphi) \times \mathsf{S}^R) \cap W^{\otimes}$ if $\varphi \in S_{\mathsf{EMEDL}}$, and

(b) $\varnothing$ if otherwise, and

(v) $V^{\otimes}$ is the function that assigns a subset $V^{\otimes}(p)$ of $W^{\otimes}$ to each proposition

letter $p \in \mathsf{P}$ such that $V^{\otimes}(p) = \{(w, \mathsf{s}) \in W^{\otimes} \mid w \in V(p)\}$. □

Note that the clauses (iii)-(b) and (iii)-(c) are based on the clauses on $\mathcal{M}_{\mathrm{Command}_{(i,j)\psi}}$ and $\mathcal{M}_{\mathrm{Request}_{(i,j)\psi}}$ in Definition 11, respectively. They define how com and req work. According to (iii)-(b), the update by M^R cuts every link of deontic accessibility for $\mathsf{O}_{(a,c,c)}$ from (w, com) to (v, com) for any (w, v) in $D_{(a,c,c)}$ if r does not hold in v before the update. Thus, $\mathsf{O}_{(a,c,c)}r$ holds in (w, com) for any (w, com) after the update by M^R. Similarly, according to (iii)-(c), $\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$ holds in (w, req) for any (w, req), after the update by M^R.

For the sake of safety, we require (1) φ to be in S_{EMEDL} in Clause (f) and (2) $O^{\otimes}(i, j, h, \varphi)$ to be the empty set when $\varphi \notin S_{\mathsf{EMEDL}}$. (1) and (2) falsify $\mathsf{Auth}_{(i,j,h)}\varphi$ when $\varphi \notin S_{\mathsf{EMEDL}}$. As $\mathsf{S}^R = \{\mathsf{com}, \mathsf{req}\}$, for any $\mathsf{t} \in \mathsf{S}^R$, $\mathsf{pre}(\mathsf{t}) \in S_{\mathsf{EMEDL}}$.

As $\sim^{\mathcal{M}}_i$ and $\sim^{R}_i$ are equivalence relations, $\sim^{\otimes}_i$ is also an equivalence relation. Therefore, $\mathcal{M} \otimes \mathsf{M}^R$ is an $\mathcal{L}_{\mathsf{EMEDL}}$-model.

In order to analyze Example 3, we also need a model-world pair that is suitable for representing the initial situation of the event $(\mathsf{M}^R, \mathsf{req})$. Suppose $(\mathcal{M}^R, w_0)$ such that $\mathcal{M}^R = (W, E, D, O, V)$ and $w_0 \in W$ is such a pair. For the sake of simplicity, let us suppose that $O(c, a, \mathrm{g}, r) = W$. This means that in every world of $\mathcal{M}^R$, c is in a position to perform both req and com.

If $(\mathcal{M}^R, w_0)$ is to be suitable for the initial situation of the event M^R, we must have the following.

$$\mathcal{M}^R, w_0 \vDash_{\mathsf{DAM}^-} \neg\mathsf{K}_i r \wedge \neg\mathsf{K}_i\neg r \text{ for any } i \in \{a, c, d\}. \tag{1}$$
$$\mathcal{M}^R, w_0 \vDash_{\mathsf{DAM}^-} \neg\mathsf{O}_{(a,c,a)} r \wedge \neg\mathsf{O}_{(a,c,a)}\neg r. \tag{2}$$
$$\mathcal{M}^R, w_0 \vDash_{\mathsf{DAM}^-} \neg\mathsf{O}_{(a,c,c)} r \wedge \neg\mathsf{O}_{(a,c,c)}\neg r. \tag{3}$$
$$\mathcal{M}^R, w_0 \vDash_{\mathsf{DAM}^-} \neg\mathsf{O}_{(a,c,c)}(\mathrm{K}_c\mathsf{O}_{(a,c,a)} r \vee \mathrm{K}_c\neg\mathsf{O}_{(a,c,a)} r). \tag{4}$$

(1) means that no one knows whether r holds or not in ($\mathcal{M}^R$, w_0). (2) means that a has neither committed himself to seeing to it that r nor to seeing to it that $\neg r$ in ($\mathcal{M}^R$, w_0). Moreover, (3) means that it is not obligatory for a towards c due to c to see to it that r, nor is it so for a to see to it that $\neg r$ in ($\mathcal{M}^R$, w_0). Lastly, (4) means that it is not obligatory for a to let c know whether he commits himself to seeing to it that r or not in ($\mathcal{M}^R$, w_0). These conditions characterize what holds before the event (M^R, req).

In addition, if ($\mathcal{M}^R \otimes \mathsf{M}^R$, ($w_0$, req)) is to be suitable for representing how the event (M^R, req) changes the situation ($\mathcal{M}^R$, w_0), we must have the following.

$$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)} r). \tag{5}$$
$$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \mathsf{K}_a\mathsf{O}(a,c,c)(\mathsf{K}_c\mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)} r). \tag{6}$$
$$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \mathsf{K}_c\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)} r). \tag{7}$$
$$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \neg\mathsf{K}_d\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)} r). \tag{8}$$

(5) means that it is obligatory for a to let c know whether he commits himself to seeing to it that r or not in the final situation of (M^R, req). Thus, (6), (7), and (8) jointly mean that a and c, but not d, know what has happened in the final situation of (M^R, req). So, if (6), (7), and (8) are shown to hold, we can say that uptake is secured, although d remains uncertain.

Note that for any $\mathcal{L}_{\mathsf{EMEDL}}$-model $\mathcal{M}$ for any worlds v and u of $\mathcal{M}$, ((v, com), (u, com)) $\in D^{\otimes}_{(a,c,c)}$ implies

$$\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{com}) \vDash_{\mathsf{DAM}^-} r.$$

But this does not imply

$$\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{com}) \vDash_{\mathsf{DAM}^-} (\mathsf{K}_c\mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)} r).$$

since r does not imply ($\mathsf{K}_c\mathsf{O}(a,c,a)r \vee \mathsf{K}_c\neg\mathsf{O}(a,c,a)r$). So, if there are worlds w, v, and u in $\mathcal{M}$ such that (w, req) $\sim^{\otimes}_d$ (v, com), ((v, com), (u, com)) $\in D^{\otimes}_{(a,c,c)}$ and

$$\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{com}) \vDash_{\mathsf{DAM}^-} \neg(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r).$$

we have

$$\mathcal{M} \otimes \mathsf{M}^R, (w, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \neg\mathsf{K}_d\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r).$$

We now build such a model $\mathcal{M}^R$ and show that we have (8) as well as (1)–(7).

Definition 18 (The model $\mathcal{M}^R$) Given the language $\mathcal{L}_{\mathsf{DAM}^-}$ based on the same countably infinite set of proposition letters P with $r \in \mathsf{P}$, the same finite set of agents $A = \{a, c, d\}$, and the same finite set of organizations H with $g \in H$, let $\mathcal{M}^R = (W, E, D, O, V)$ be an $\mathcal{L}_{\mathsf{EMEDL}}$-model satisfying the following conditions.

(i) $W = \{w_0, v_0, v_1, v_2\}$,

(ii) $E = \{\sim_i \mid i \in A\}$ such that $\sim_a = \sim_c = \sim_d =$
$\{(w_0, v_1), (v_1, w_0), (w_0, v_2), (v_2, w_0), (v_1, v_2), (v_2, v_1)\} \cup \{(w, v) \in W \times W \mid w = v\}$,

(iii) $D = \{D_{(a,c,a)}, D_{(a,c,c)}\}$ such that
$D_{(a,c,a)} = \{(w_0, v_0), (w_0, v_1), (w_0, v_2), (v_0, v_0), (v_1, v_1), (v_2, v_2)\}$ and
$D_{(a,c,c)} = \{(w_0, v_0), (w_0, v_1), (w_0, v_2)\}$,

(iv) O is a function that assigns a subset $O(i, j, h, \varphi)$ of W to each
$(i, j, h, \varphi) \in A \times A \times H \times S_{\mathsf{EMEDL}}$ such that $O(c, a, \mathrm{g}, r) = W$, and

(v) V is a function that assigns a subset $V(p)$ of W to each proposition letter $p \in \mathsf{P}$ such that
$V(r) = \{w_0, v_2\}$. □

Note that $\sim_i$ is required to be an equivalence relation. We use 'w' and 'v' as metavariables ranging over W and use 'w_0', 'v_0', 'v_1', and 'v_2' as the names of the four worlds in W. For any accessibility relation R, we say v is R-accessible from w if $(w, v) \in R$.

We now show $(\mathcal{M}^R, w_0)$ is suitable for representing the initial situation of the event $(\mathsf{M}^R, \mathsf{req})$. As $V(r) = \{w_0, v_2\}$, we have

$$\mathcal{M}^R, v_1 \vDash_{\mathsf{DAM}^-} \neg r \tag{9}$$

and

$$\mathcal{M}^R, v_2 \vDash_{\mathsf{DAM}^-} r. \tag{10}$$

As v_1 and v_2 are $\sim_i$-accessible from w_0 for each $i \in A$, we thus have

$$\mathcal{M}^R, w_0 \vDash_{\mathsf{DAM}^-} \neg\mathsf{K}_i r \wedge \neg\mathsf{K}_i\neg r \text{ for any } i \in \{a, c, d\}. \tag{1}$$

Thus, no one knows whether r holds or not in the situation ($\mathcal{M}^R, w_0$).

As v_1 and v_2 are both $D_{(a,c,a)}$-accessible and $D_{(a,c,c)}$-accessible from w_0, we have

$$\mathcal{M}^R, w_0 \vDash_{\mathsf{DAM}^-} \neg\mathsf{O}_{(a,c,a)} r \wedge \neg\mathsf{O}_{(a,c,a)}\neg r \tag{2}$$

and

$$\mathcal{M}^R, w_0 \vDash_{\mathsf{DAM}^-} \neg\mathsf{O}_{(a,c,c)} r \wedge \neg\mathsf{O}_{(a,c,c)}\neg r. \tag{3}$$

Thus, a has neither committed himself to seeing to it that r nor to seeing to it that $\neg r$ in ($\mathcal{M}^R, w_0$). Moreover, it is not obligatory for him towards c due to c to see to it that r, nor is it so for him to see to it that $\neg r$ in that situation.

Then, consider (4), i.e.,

$$\mathcal{M}^R, w_0 \vDash_{\mathsf{DAM}^-} \neg\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)} r). \tag{4}$$

As v_1 is $D_{(a,c,a)}$-accessible from v_1 and v_2 is the only world $D_{(a,c,a)}$-accessible from v_2, (9) and (10) imply

$$\mathcal{M}^R, v_1 \vDash_{\mathsf{DAM}^-} \neg\mathsf{O}_{(a,c,a)} r \tag{11}$$

and

$$\mathcal{M}^R, v_2 \vDash_{\mathsf{DAM}^-} \mathsf{O}_{(a,c,a)} r, \tag{12}$$

respectively. As v_1 and v_2 are $\sim_c$-accessible from v_1, we have

$$\mathcal{M}^R, v_1 \vDash_{\mathsf{DAM}^-} \neg(\mathsf{K}_c\mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)} r). \tag{13}$$

As v_1 is $D_{(a,c,c)}$-accessible from w_0, we thus have

$$\mathcal{M}^R, w_0 \vDash_{\mathsf{DAM}^-} \neg\mathrm{O}(a,c,c)(\mathsf{K}_c\mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)} r). \tag{4}$$

Thus, it is not obligatory for a to let c know whether he commits himself to seeing to it that r or not in ($\mathcal{M}^R, w_0$).

As $\mathsf{pre}^R(\mathsf{req}) = \top$, $\mathsf{pre}^R(\mathsf{com}) = \mathsf{Auth}_{(c,a,\mathrm{g})}r$, and $O(c, a, \mathrm{g}, r) = W$, we have, for any $w \in W$,

$$\mathcal{M}^R, w \vDash_{\mathsf{DAM}^-} \mathsf{pre}^R(\mathsf{req}) \tag{14}$$

and

$$\mathcal{M}^R, w \vDash_{\mathsf{DAM}^-} \mathsf{pre}^R(\mathsf{com}). \tag{15}$$

This means that it is possible for c to request a to see to it that r as well as to command a to see to it that r in every $w \in W$, and this in turn means that it is possible for c to do either of these acts in the situation represented by $(\mathcal{M}^R, w_0)$. Thus, $(\mathcal{M}^R, w_0)$ is suitable for representing the initial situation of $(\mathsf{M}^R, \mathsf{req})$.

Now let us examine how $(\mathcal{M}^R, w_0)$ is changed when updated by $(\mathsf{M}^R, \mathsf{req})$.[17]

Fact 2 ($\mathcal{M}^R \otimes \mathsf{M}^R$) The result $\mathcal{M}^R \otimes \mathsf{M}^R$ of updating $\mathcal{M}^R$ with M^R is a tuple

$$(W^\otimes, E^\otimes, D^\otimes, O^\otimes, V^\otimes)$$

satisfying the following conditions.

(i) $W^\otimes = \{(w, \mathsf{s}) \in W \times \mathsf{S}^R \mid \mathcal{M}^R, w \vDash_{\mathsf{DAM}^-} \mathsf{pre}^R(\mathsf{s})\}$,

(ii) $E^\otimes = \{\sim_i^\otimes \mid i \in A\}$ such that $((w, \mathsf{s}), (v, \mathsf{t})) \in \sim_i^\otimes$ iff $w \sim_i v$ and $\mathsf{s} \sim_i^R \mathsf{t}$ for any $(w, \mathsf{s}), (v, \mathsf{t}) \in W^\otimes$,

(iii) $D^\otimes = \{D^\otimes_{(i,j,k)} \mid, i, j, k \in A\}$ such that

$D^\otimes_{(i,j,k)} = \{((w,\mathsf{s}),(v,\mathsf{t})) \in W^\otimes \times W^\otimes \mid (w,v) \in D^{\mathsf{s}}_{(i,j,k)}$ and $\mathsf{s} = \mathsf{t}\}$, where

(a) $D^{\mathsf{s}}_{(i,j,k)} = D_{(i,j,k)}$ if $(i, j, k) \neq (a, c, c)$,

(b) $D^{\mathsf{s}}_{(a,c,c)} = \{(w, v) \in D_{(a,c,c)} \mid \mathcal{M}^R, v \vDash_{\mathsf{DAM}^-} r\}$ if $\mathsf{s} = \mathsf{com}$, and

(c) $D^{\mathsf{s}}_{(a,c,c)} = \{(w, v) \in D_{(a,c,c)} \mid \mathcal{M}^R, v \vDash_{\mathsf{DAM}^-} \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r\}$ if $\mathsf{s} = \mathsf{req}$,

(iv) $O^\otimes$ is a function that assigns a subset $O^\otimes(i, j, h, \varphi)$ of $W^\otimes$

to each $(i, j, h, \varphi) \in A \times A \times H \times S_{\mathsf{DAM}^-}$ such that $O^\otimes(i, j, h, \varphi) =$

(a) $\{(w, \mathsf{s}) \in W^\otimes \mid w \in O(i, j, h, \varphi)\}$ if $\varphi \in S_{\mathsf{EMEDL}}$, and

(b) $\varnothing$ if otherwise, and

(v) $V^\otimes$ is a function that assigns a subset $V^\otimes(p)$ of $W^\otimes$ to each proposition letter $p \in \mathsf{P}$ such that $V^\otimes(p) = \{(w, \mathsf{s}) \in W^\otimes \mid w \in V(p)\}$. □

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17 $(\mathsf{M}^R, \mathsf{req})$ is defined in Definition 16 on Pages 20–21.

Obviously, $\mathcal{M}^R \otimes \mathsf{M}^R$ is an instance of $\mathcal{M} \otimes \mathsf{M}^R$ defined in Definition 17.

As $O(c, a, g, r) = W$ by Definition 18, $O^{\otimes}(c, a, g, r) = W^{\otimes}$. As (14) and (15) jointly guarantee that $W^{\otimes} = W \times \mathsf{S}^R$, we get $(\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}))$ by updating $(\mathcal{M}^R, w_0)$ with $(\mathsf{M}^R, \mathsf{req})$.

As $\sim_i$ and $\sim_i^R$ are required to be equivalence relations, $\sim_i^{\otimes}$ is required to be an equivalence relation. Thus, $\mathcal{M}^R \otimes \mathsf{M}^R$ is an $\mathcal{L}_{\mathsf{EMEDL}}$-model.

Now, let us move on to (5)–(8). First, consider (5), i.e.,

$$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{5}$$

Here we need to examine which worlds are $D^{\otimes}_{(a,c,c)}$-accessible from (w_0, req). First, consider (w_0, req) itself. For any worlds w and v of $\mathcal{M}^R$, (v, req) is $D^{\otimes}_{(a,c,c)}$-accessible from (w, req) iff v is $D_{(a,c,c)}$-accessible from w and $\mathcal{M}^R, v \vDash_{\mathsf{DAM}^-} \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r$. This means that (w_0, req) is not $D^{\otimes}_{(a,c,c)}$-accessible from (w_0, req) since w_0 is not $D_{(a,c,c)}$-accessible from w_0. Then, consider (v_0, req). As $v_0 \notin V(r)$ and v_0 is $D_{(a,c,a)}$-accessible from v_0, we have

$$\mathcal{M}, v_0 \vDash_{\mathsf{DAM}^-} \neg\mathsf{O}_{(a,c,a)}r. \tag{16}$$

Since v_0 is the only world $\sim_c$-accessible from v_0, we have

$$\mathcal{M}, v_0 \vDash_{\mathsf{DAM}^-} \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r. \tag{17}$$

Since this implies

$$\mathcal{M}, v_0 \vDash_{\mathsf{DAM}^-} \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r, \tag{18}$$

(v_0, req) is $D^{\otimes}_{(a,c,c)}$-accessible from (w_0, req), as v_0 is $D_{(a,c,c)}$-accessible from w_0 in $\mathcal{M}^R$. Then consider (v_1, req). Note that we have (11) and (12). As v_1 and v_2 are $\sim_c$-accessible from v_1, we have

$$\mathcal{M}, v_1 \vDash_{\mathsf{DAM}^-} \neg(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{19}$$

This implies that (v_1, req) is not $D^{\otimes}_{(a,c,c)}$-accessible from (w_0, req). Then, finally, consider (v_2, req). As v_1 and v_2 are $\sim_c$-accessible from v_2, (11) and (12) jointly imply

$$\mathcal{M}, v_2 \vDash_{\mathsf{DAM}^-} \neg(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{20}$$

This, in turn, implies that (v_2, req) is not $D^{\otimes}_{(a,c,c)}$-accessible from (w_0, req). Since only w_0, v_0, v_1, and v_2 are $D_{(a,c,c)}$-accessible from w_0 in $\mathcal{M}^R$, only (v_0, req) is $D^{\otimes}_{(a,c,c)}$-accessible from (w_0, req) in $\mathcal{M}^R \otimes \mathsf{M}^R$.

Now consider what we can say about (v_0, req). Since $V(r) = \{w_0, v_2\}$, $(v_0, \mathsf{req}) \notin V^{\otimes}(r)$. As (v_0, req) is $D^{\otimes}_{(a,c,a)}$-accessible from (v_0, req), we have

$$\mathcal{M}^R \otimes \mathsf{M}^R, (v_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \neg\mathsf{O}_{(a,c,a)}r, \tag{21}$$

As (v_0, req) is the only world $\sim^{\otimes}_c$-accessible from (v_0, req), we have

$$\mathcal{M}^R \otimes \mathsf{M}^R, (v_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r. \tag{22}$$

This implies

$$\mathcal{M}^R \otimes \mathsf{M}^R, (v_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r. \tag{23}$$

Since (v_0, req) is the only world $D^{\otimes}_{(a,c,c)}$-accessible from (w_0, req), we have

$$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{5}$$

Now, this means that we also have

$$\mathcal{M}^R, w_0 \vDash_{\mathsf{DAM}^-} [(\mathsf{M}^R, \mathsf{req})]\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{24}$$

Since r is $\mathsf{O}_{(a,c,c)}$-free, this is an instance of the DAM^- analogue of (RUGO) of DMEDL. Although we have only seen that the formula $[(\mathsf{M}^R, \mathsf{req})]\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$ is satisfiable here, it is not difficult to see that it is valid. For any $\mathcal{L}_{\mathsf{EMEDL}}$-model $\mathcal{M}$, (v, req) is $D^{\otimes}_{(a,c,c)}$-accessible from (w, req) in $\mathcal{M} \otimes \mathsf{M}^R$ iff $\mathcal{M}, v \vDash_{\mathsf{DAM}^-} \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r$. We will not go into the formal proof of the validity of this DAM^- analogue of (RUGO) here, however, since the most difficult target of this paper still remains to be analyzed. It is the scenario in which uptake is not secured.

Before that, however, let us examine (6), (7), and (8). Note that $\sim_a^R = \sim_c^R = \{(\mathsf{com},\mathsf{com}),(\mathsf{req},\mathsf{req})\}$. This means that for any $i \in \{a, c\}$, $\sim_i^{\otimes} = \{((w, \mathsf{com}), (v, \mathsf{com})) \mid (w, v) \in \sim_i\} \cup \{((w, \mathsf{req}), (v, \mathsf{req})) \mid (w, v) \in \sim_i\}$.

Now consider (6), i.e.,

$$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \mathsf{K}_a\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{6}$$

Note that w_0, v_1, and v_2 are the only worlds $\sim_a$-accessible from w_0 in $\mathcal{M}^R$. This means that (w_0, req), (v_1, req), and (v_2, req) are the only worlds $\sim_a^{\otimes}$-accessible from (w_0, req) in $\mathcal{M}^R \otimes \mathsf{M}^R$.

Now, no world is $D_{(a,c,c)}$-accessible from v_1 in $\mathcal{M}^R$. So, no world is $D^{\otimes}_{(a,c,c)}$-accessible from (v_1, req) in $\mathcal{M}^R \otimes \mathsf{M}^R$. Thus, vacuously, we have

$$\mathcal{M}^R \otimes \mathsf{M}^R, (v_1, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{25}$$

Similarly, we have

$$\mathcal{M}^R \otimes \mathsf{M}^R, (v_2, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{26}$$

Then, (5), (25), and (26) jointly imply (6).

Since (w_0, req), (v_0, req), and (v_1, req) are also the only worlds $\sim_c^{\otimes}$-accessible from (w_0, req), a reasoning similar to the one we have just presented establishes (7), i.e.,

$$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \mathsf{K}_c\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{7}$$

Now, (6) implies

$$\mathcal{M}^R, w_0 \vDash_{\mathsf{DAM}^-} [(\mathsf{M}^R, \mathsf{req})]\mathsf{K}_a\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{27}$$

This is an instance of the DAM^- analogue of the two-option version of (RUGU) of DMEDL. Note that a remark similar to the one about (24) also applies to (27).

Then, finally, consider (8), i.e.,

$$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \neg\mathsf{K}_d\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{8}$$

Note that in addition to (w_0, req), (v_1, req), and (v_2, req), (w_0, com), (v_1, com), and (v_2, com) are $\sim_d^{\otimes}$-accessible from (w_0, req) in $\mathcal{M}^R \otimes \mathsf{M}^R$ as we have $w_0 \sim_d w_0$, $w_1 \sim_d v_1$, $w_0 \sim_d v_2$ and req $\sim_d^R$ com. As $v_2 \in V(r)$, (v_2, com) $\in V^{\otimes}(r)$. Thus, we have

$$\mathcal{M}^R \otimes \mathsf{M}^R, (v_2, \mathsf{com}) \vDash_{\mathsf{DAM}^-} r. \tag{28}$$

(28) implies that (v_2, com) is $D^{\otimes}_{(a,c,c)}$-accessible from (w_0, com), as v_2 is $D_{(a,c,c)}$-accessible from w_0 in $\mathcal{M}^R$.

As v_1 is $D_{(a,c,a)}$-accessible from v_1 and $v_1 \notin V(r)$ in $\mathcal{M}^R$, (v_1, com) is $D^{\otimes}_{(a,c,a)}$-accessible from (v_1, com) and (v_1, com) $\notin V^{\otimes}(r)$. So, we have

$$\mathcal{M}^R \otimes \mathsf{M}^R, (v_1, \mathsf{com}) \vDash_{\mathsf{DAM}^-} \neg\mathsf{O}_{(a,c,a)}r. \tag{29}$$

As v_2 is the only world $D_{(a,c,a)}$-accessible from v_2, (v_2, com) is the only world $D^{\otimes}_{(a,c,a)}$-accessible from (v_2, com), and so (28) implies

$$\mathcal{M}^R \otimes \mathsf{M}^R, (v_2, \mathsf{com}) \vDash_{\mathsf{DAM}^-} \mathsf{O}_{(a,c,a)}r. \tag{30}$$

As v_1 and v_2 are $\sim_c$-accessible from v_2 in $\mathcal{M}^R$, (v_1, com) and (v_2, com) are $\sim_c^{\otimes}$-accessible from (v_2, com). So, we have

$$\mathcal{M}^R \otimes \mathsf{M}^R, (v_2, \mathsf{com}) \vDash_{\mathsf{DAM}^-} \neg(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{31}$$

As (v_2, com) is $D^{\otimes}_{(a,c,c)}$-accessible from (w_0, com), we have

$$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{com}) \vDash_{\mathsf{DAM}^-} \neg\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{32}$$

As req $\sim_d^R$ com, (w_0, com) is $\sim_d^{\otimes}$-accessible from (w_0, req). So, we have

(8) $$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \neg\mathsf{K}_d\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{8}$$

This enables us to avoid the DAM^- analogue of (SRUGU) of the two-option version of DMEDL. Thus, the method of deontic product update works fine in analyzing the situation in Example

3. (6) means that a knows what he is to do, but (8) shows that d is uncertain what has happend in ($\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req})$); uptake is secured in spite of d's uncertainty.

5. When Uptake Is Not Secured

Can the deontic product update be applied to the situations in Example 1?

In that example, a was uncertain whether the guru had requested or commanded in the situation just after her act of saying "I would be happy if you could lead our group in that demonstration." But if so, uptake was not yet secured. As the securing of uptake is necessary for performing an illocutionary act, the guru had not succeeded in performing an act of requesting of the form Request$_{(c,a)}r$. This is what we have seen in Example 1.

Since uptake had not been secured in that situation, the guru must also have been uncertain whether she had succeeded in making a request or not even if she had intended to make a request when she had said "I would be happy if you could lead our group in that demonstration."

Now, it might seem easy to construct a pointed $\mathcal{L}_{\mathsf{EMEDL}}$-model, say ($\mathcal{M}^{R*}, w_0$), that is suitable for representing the situation before the guru's act of saying "I would be happy if you could lead our group in that demonstration." We can just drop d and suppose a and c to be like d in Example 3.

So, let us imagine an action model $\mathsf{M}^{R*} = \{\mathsf{S}^*, \sim^*, \mathsf{pre}^*\}$ such that $\mathsf{S}^* = \{\mathsf{req}, \mathsf{com}\}$, $\sim^* = \{\sim^*_a, \sim^*_c\}$, $\sim^*_a = \sim^*_c = \mathsf{S}^* \times \mathsf{S}^*$, $\mathsf{pre}^*(\mathsf{com}) = \mathsf{Auth}_{(c,a,g)}r$, and $\mathsf{pre}^*(\mathsf{req}) = \top$. Let $\mathcal{M}^{R*}$ be a model obtained from $\mathcal{M}^R$ by dropping d. Then, by steps similar to those that show (8) in Section 4, we would get, for any $i \in \{a, c\}$,

$$\mathcal{M}^{R*} \otimes \mathsf{M}^{R*}, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \neg \mathsf{K}_i \mathsf{O}_{(a,c,c)}(\mathsf{K}_c \mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)}r). \tag{33}$$

Thus, uptake is not secured.

This model, however, is not exactly what we want, for we would also have

$$\mathcal{M}^{R*} \otimes \mathsf{M}^{R*}, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \mathsf{O}_{(a,c,c)}(\mathsf{K}_c \mathsf{O}_{(a,c,a)}r) \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)}r). \tag{34}$$

$\mathcal{M}^{R*} \otimes \mathsf{M}^{R*}$ is the model obtained by updating $\mathcal{M}^{R*}$ with M^{R*} and the world (w_0, req) is the world to be obtained when req is performed in ($\mathcal{M}^{R*}, w_0$). Thus, ($\mathcal{M}^{R*} \otimes \mathsf{M}^{R*}, (w_0, \mathsf{req})$) represents the situation that would result if c had performed req (an act of requesting of the form Request$_{(c,a)}r$), but (33) means that a was not able to tell which of req and com had been performed in $(\mathcal{M}^{R*}, w_0)$. It is as if req had really been performed without being understood.

If the addressee did not understand the force of the utterance, however, the intended illocutionary act had not yet been completed. But then, what had actually happened?

One possible move here is to say that the guru had actually tried to make a request but failed. A locutionary act of saying "I would be happy if you could lead our group in that demonstration" had been performed but the intended illocutionary act had not come into effect.

It may also be said that the guru had told a that she would be happy if he could lead their group in the demo. An act of telling so is an illocutionary act. Thus, she had tried to make a request by performing another illocutionary act of telling him that she would be happy if he could lead their group in the demo. Understood in this way, her act can be considered as an instance of the so-called indirect speech acts. But even if a understood that she had told him that she would be happy if he could lead their group in the demo, her attempted act of requesting would not yet be completed unless a understood her utterance to be a request.

Here it is of some help to compare Example 1 with one of its variants briefly discussed in Section 2. In that scenario, a has neither been invited to the conference in São Paulo nor has any other prior commitments that would prevent him from joining the demonstration. Moreover, he is willing to help the guru, and her utterance enables him to learn that she would be happy if he could lead the group in the demonstration. In such a situation, he may gladly and immediately say that he will lead the group without bothering himself about whether she has requested or commanded. If he does so, she may gladly accept his offer.[18]

Now, the situation a is in in this scenario is very different from the situation he was in in Example 1. Nevertheless, one feature is shared by them. Let s be the proposition that c will be happy. Then, what she had told a (in the informative sense) may be represented by the proposition that $\neg(r \wedge \neg s)$, and we can say that c had not only told a that $\neg(r \wedge \neg s)$, but also succeeded in getting a to learn that $\neg(r \wedge \neg s)$ in Example 1 just as c does in the variant scenario.

An interesting possibility here is to consider an action model with just one action point, say gl, which names c's act of getting a to learn that $\neg(r \wedge \neg s)$. Since this act involves the real effect of a's learning (coming to know) that $\neg(r \wedge \neg s)$, it is a perlocutionary act. It can be modeled as a so-called 'public announcement' in the community whose members are just a and c. So,

18 Such examples may be supposed to support Cohen & Levesque's thesis that illocutionary force recognition is unnecessary (1988, pp. 2–4). However, there are cases where illocutionary force recognition is important not only for theorists but also for the agents involved. This, of course, is not meant to deny the fact that there are many cases where implicitness is important. But in Example 1, whether he had the option of refusal or not was important for a. As it was unclear, he wished to know whether the guru had meant to give a command or to make a request.

following van Ditmarsch, van der Hoek & Kooi (2007, p. 150), we suppose $\neg(r \wedge \neg s)$ to be the precondition for gl.[19]

Let M^G be the action model with just one action point gl. As gl is an act that affects epistemic states of agents, an update by M^G is an epistemic product update. The situation where the act gl is performed, however, has deontic aspects as well as epistemic aspects. Thus, the language suitable for dealing with Example 1 and the product update by M^G need to be something more than $\mathcal{L}_{\mathsf{AM}^-}$ and $\mathcal{L}_{\mathsf{DAM}^-}$.

Definition 19 (The language $\mathcal{L}_{\mathsf{EDAM}^-}$) Given a countably infinite set P of proposition letters such that $r, s \in \mathsf{P}$, a finite set of agents A such that $A = \{a, c\}$, and a finite set of organizations H such that $g \in H$, with p ranging over P, i, j, k ranging over A, and h ranging over H, the language $\mathcal{L}_{\mathsf{EDAM}^-}$ of epistemic action model logic with deontic modalities EDAM^- is given by

$$
\begin{aligned}
\varphi\ &::= p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid \mathsf{K}_i\varphi \mid \mathsf{O}_{(i,j,k)}\varphi \mid \mathsf{Auth}_{(i,j,h)}\varphi \mid [\alpha]\varphi,\\
\alpha\ &::= (\mathsf{M}, \mathsf{s}),
\end{aligned}
$$

where (M, s) is a pointed epistemic action model with a finite set of action points S such that $\mathsf{s} \in \mathsf{S}$, and for any $\mathsf{t} \in \mathsf{S}$, $\mathsf{pre}(\mathsf{t})$ is a formula of this language that has already been constructed in a previous stage of the inductively defined hierarchy. The set of all well-formed formulas (sentences) of $\mathcal{L}_{\mathsf{EDAM}^-}$ is denoted by S_{EDAM^-}. □

Note that (M, s) here is a pointed epistemic action model similar to the one we have seen in Section 3. Although Definition 19 may look similar to Definition 15, (M, s) mentioned in Definition 15 is a pointed deontic action model. Thus, $\mathcal{L}_{\mathsf{EDAM}^-}$ is different from $\mathcal{L}_{\mathsf{DAM}^-}$.[20]

$\mathcal{L}_{\mathsf{EDAM}^-}$ can be used to characterize the situations in Example 1. First, we need a few more definitions.

19 Although it may sound surprising, the way a 'public announcement' is treated in PAL (Public Announcement Logic) shows that it is modeled as a perlocutionary act. In PAL, a truthful public announcement that φ is supposed to have the effect of getting everyone to know that φ if the operator K_i for any agent i does not occur in φ. This supposition seems to be too strong if the "public announcement" is understood as an illocutionary act. In real-life situations, some of the addressees may remain unconvinced even if a truthful public announcement is made. For more on PAL, see van Ditmarsch, van der Hoek & Kooi (2007, Chapter 4).

20 It may be interesting to consider an action model which includes gl, com, and req, but we will not pursue it in this paper.

Definition 20 (The pointed action model (M^G, gl)) Given the language $\mathcal{L}_{\mathsf{EDAM}^-}$ based on a countably infinite set P of proposition letters such that $r, s \in \mathsf{P}$, a finite set of agents $A = \{a, c\}$, and a finite set of organizations H such that $g \in H$, the pointed action model that represents an event including only c's act gl is the pair (M^G, gl), where M^G is a tuple

$$(\mathsf{S}^G, \{\sim_a^G, \sim_c^G\}, \mathsf{pre}^G)$$

such that (i) $\mathsf{S}^G = \{\mathsf{gl}\}$, (ii) $\sim_a^G = \sim_c^G = \{(\mathsf{gl}, \mathsf{gl})\}$, and (iii) $\mathsf{pre}^G(\mathsf{gl}) = \neg(r \wedge \neg s)$. □

Definition 21 ($\vDash_{\mathsf{EDAM}}$) Given the language $\mathcal{L}_{\mathsf{EDAM}^-}$ based on the same countably infinite set of proposition letters P with $r, s \in \mathsf{P}$, the same finite set of agents $A = \{a, c\}$, the same finite set of organizations H such that $g \in H$, and the action model M^G, let $\mathcal{M}$ be an $\mathcal{L}_{\mathsf{EMEDL}}$-model (W, E, D, O, V) such that $E = \{\sim_i \mid i \in A\}$, $D = \{D^{\mathcal{M}}_{(i,j,k)} \mid, i, j, k \in A\}$, and $O(c, a, \mathrm{g}, r) = W$. If $w \in W$, $\mathsf{s} \in \mathsf{S}^G$, $p \in \mathsf{P}$, $\varphi, \psi \in S_{\mathsf{EDAM}^-}$, $i, j, k \in A$, and $h \in H$, then the satisfaction relation $\vDash_{\mathsf{EDAM}^-}$ is defined as follows.

(a) $\mathcal{M}, w \vDash_{\mathsf{EDAM}^-} p$ iff $w \in V(p)$,
(b) $\mathcal{M}, w \vDash_{\mathsf{EDAM}^-} \neg\varphi$ iff $\mathcal{M}, w \nvDash_{\mathsf{EDAM}^-} \varphi$,
(c) $\mathcal{M}, w \vDash_{\mathsf{EDAM}^-} (\varphi \wedge \psi)$ iff $\mathcal{M}, w \vDash_{\mathsf{EDAM}^-} \varphi$ and $\mathcal{M}, w \vDash_{\mathsf{EDAM}^-} \psi$,
(d) $\mathcal{M}, w \vDash_{\mathsf{EDAM}^-} \mathsf{K}_i\varphi$ iff for every v such that $(w, v) \in \sim_i$, $\mathcal{M}, v \vDash_{\mathsf{EDAM}^-} \varphi$,
(e) $\mathcal{M}, w \vDash_{\mathsf{EDAM}^-} \mathsf{O}_{(i,j,k)}\varphi$ iff for every v such that $(w, v) \in D_{(i,j,k)}$, $\mathcal{M}, v \vDash_{\mathsf{EDAM}^-} \varphi$,
(f) $\mathcal{M}, w \vDash_{\mathsf{EDAM}^-} \mathsf{Auth}_{(i,j,h)}\varphi$ iff $\varphi \in S_{\mathsf{EMEDL}}$ and $w \in O(i, j, h, \varphi)$, and
(g) $\mathcal{M}, w \vDash_{\mathsf{EDAM}^-} [(\mathsf{M}^G, \mathsf{s})]\varphi$ iff $\mathcal{M}, w \vDash_{\mathsf{EDAM}^-} \mathsf{pre}^G(\mathsf{s})$ implies $\mathcal{M} \otimes \mathsf{M}^G, (w, \mathsf{s}) \vDash_{\mathsf{EDAM}^-} \varphi$,

where $\mathcal{M} \otimes \mathsf{M}^G$ is a tuple

$$(W^{\otimes}, E^{\otimes}, D^{\otimes}, O^{\otimes}, V^{\otimes})$$

satisfying the following conditions.
(i) $W^{\otimes} = \{(w, \mathsf{s}) \in W \times \mathsf{S}^G \mid \mathcal{M}, w \vDash_{\mathsf{EDAM}^-} \mathsf{pre}^G(\mathsf{s})\}$,
(ii) $E_i^{\otimes} = \{\sim_i^{\otimes} \mid i \in A\}$ such that
$(w, \mathsf{s}) \sim_i^{\otimes} (v, \mathsf{t})$ iff $w \sim_i v$ and $\mathsf{s} \sim_i^G \mathsf{t}$ for any $(w, \mathsf{s}), (v, \mathsf{t}) \in W^{\otimes}$,
(iii) $D^{\otimes} = \{D^{\otimes}_{(i,j,k)} \mid, i, j, k \in A\}$ such that
$D^{\otimes}_{(i,j,k)} = \{((w, \mathsf{s}), (v, \mathsf{t})) \in w^{\otimes} \times w^{\otimes} \mid (w, v) \in D_{(i,j,k)}$ and $\mathsf{s} = \mathsf{t}\}$,

(iv) $O^{\otimes}$ is a function that assigns a subset $O^{\otimes}(i, j, h, \varphi)$ of $W^{\otimes}$

to each $(i, j, h, \varphi) \in A \times A \times H \times S_{\mathsf{EDAM}^-}$ such that $O^{\otimes}(i, j, h, \varphi) =$

(a) $(O(i, j, h, \varphi) \times \mathsf{S}^G) \cap W^{\otimes}$ if $\varphi \in S_{\mathsf{EMEDL}}$, and

(b) $\emptyset$ if otherwise, and

(v) $V^{\otimes}$ is the function that assigns a subset $V^{\otimes}(p)$ of $W^{\otimes}$ to each proposition letter $p \in \mathrm{P}$ such that $V^{\otimes}(p) = \{(w, \mathsf{s}) \in W^{\otimes} \mid w \in V(p)\}$. □

As $\mathcal{M}$ is an $\mathcal{L}_{\mathsf{EMEDL}}$-model, $\sim_i$ is required to be an equivalence relation. As $\sim_i^G$ is also required to be an equivalence relation, $\sim_i^{\otimes}$ is required to be an equivalence relation for any $i \in A$. Thus, $\mathcal{M} \otimes \mathsf{M}^G$ is an $\mathcal{L}_{\mathsf{EMEDL}}$-model.

As the update by M^G eliminates worlds where $\neg(r \wedge \neg s)$ does not hold, it may possibly affect deontic accessibility relations. So, let us modify $\mathcal{M}^{R*}$ in order to get an $\mathcal{L}_{\mathsf{EMEDL}}$-model suitable for representing the initial situation of the event of the form $(\mathsf{M}^G, \mathsf{gl})$ in Example 1.

Definition 22 (The model $\mathcal{M}^G$) Given the language $\mathcal{L}_{\mathsf{EDAM}^-}$ based on the same countably infinite set P of proposition letters with $r, s \in \mathsf{P}$, the same finite set of agents $A = \{a, c\}$, and the same finite set of organizations H with $g \in H$, let $\mathcal{M}^G = (W^G, E^G, D^G, O^G, V^G)$ be an $\mathcal{L}_{\mathsf{EMEDL}}$-model satisfying the following conditions.

(i) $W^G = \{w_0, w_1, v_0, v_1, v_2\}$,

(ii) $E^G = \{\sim_i \mid i \in A\}$ such that

(a) $\sim_a = \{(w_0, w_1), (w_1, w_0), (w_0, v_1), (v_1, w_0), (w_0, v_2), (v_2, w_0), (v_1, v_2), (v_2, v_1)$
$\cup \{(w, v) \in W^G \times W^G \mid w = v\}$, and

(b) $\sim_c = \{(w_0, v_1), (v_1, w_0), (w_0, v_2), (v_2, w_0), (v_1, v_2), (v_2, v_1)\}$
$\cup \{(w, v) \in W^G \times W^G \mid w = v\}$,

(iii) $D^G = \{ D^G_{(a,c,a)}, D^G_{(a,c,c)}\}$ such that

(a) $D^G_{(a,c,a)} = \{(w_0, v_0), (w_0, v_1), (w_0, v_2), (v_0, v_0), (v_1, v_1), (v_2, v_2)\}$, and

(b) $D^G_{(a,c,c)} = \{(w_0, v_0), (w_0, v_1), (w_0, v_2)\}$,

(iv) O^G is a function that assigns a subset $O^G(i, j, h, \varphi)$ of W^G

to each $(i, j, h, \varphi) \in A \times A \times H \times S_{\mathsf{EMEDL}}$ such that $O^G(c, a, \mathrm{g}, r) = W^G$, and

(v) V^G is a function that assigns a subset $V^G(p)$ of W^G to each proposition letter

$p \in \mathsf{P}$ such that $V^G(r) = \{w_0, w_1, v_2\}$ and $V^G(s) = \{w_0, v_2\}$. □

As $\sim_i$ is required to be an equivalence relation for any $i \in A$, $\mathcal{M}^G$ is an $\mathcal{L}_{\mathsf{EMEDL}}$-model.

As $w_1 \notin V^G(s)$ but $w_1 \in V^G(r)$, we have

$$\mathcal{M}^G, w_1 \not\models_{\mathsf{EDAM}^-} \neg(r \wedge \neg s). \tag{35}$$

For any $w \in W$ such that $w \neq w_1$, however, we have

$$\mathcal{M}^G, w \models_{\mathsf{EDAM}^-} \neg(r \wedge \neg s). \tag{36}$$

This means, in particular, that we have

$$\mathcal{M}^G, w_0 \models_{\mathsf{EDAM}^-} \neg(r \wedge \neg s). \tag{37}$$

Now, it is not difficult to verify the following statements. Let $i \in \{a, c\}$.

$$\mathcal{M}^G, w_0 \models_{\mathsf{EDAM}^-} \neg\mathsf{K}_i r \wedge \neg\mathsf{K}_i \neg r. \tag{38}$$
$$\mathcal{M}^G, w_0 \models_{\mathsf{EDAM}^-} \neg\mathsf{K}_a \neg(r \wedge \neg s) \wedge \neg\mathsf{K}_a(r \wedge \neg s). \tag{39}$$
$$\mathcal{M}^G, w_0 \models_{\mathsf{EDAM}^-} \mathsf{K}_c \neg(r \wedge \neg s). \tag{40}$$
$$\mathcal{M}^G, w_0 \models_{\mathsf{EDAM}^-} \neg\mathsf{O}_{(a,c,a)} r. \tag{41}$$
$$\mathcal{M}^G, v_0 \models_{\mathsf{EDAM}^-} \neg\mathsf{O}_{(a,c,a)} r. \tag{42}$$
$$\mathcal{M}^G, v_1 \models_{\mathsf{EDAM}^-} \neg\mathsf{O}_{(a,c,a)} r. \tag{43}$$
$$\mathcal{M}^G, v_2 \models_{\mathsf{EDAM}^-} \mathsf{O}_{(a,c,a)} r. \tag{44}$$
$$\mathcal{M}^G, w_0 \models_{\mathsf{EDAM}^-} \neg(\mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg\mathsf{O}_{(a,c,a)} r). \tag{45}$$
$$\mathcal{M}^G, w_0 \models_{\mathsf{EDAM}^-} \neg\mathsf{O}_{(a,c,c)}(\mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg\mathsf{O}_{(a,c,a)} r). \tag{46}$$
$$\mathcal{M}^G, w_0 \models_{\mathsf{EDAM}^-} \neg\mathsf{K}_i \mathsf{O}_{(a,c,c)}(\mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg\mathsf{O}_{(a,c,a)} r). \tag{47}$$

Thus, $(\mathcal{M}^G, w_0)$ is suitable for representing the initial situation of the event $(\mathsf{M}^G, \mathsf{gl})$.

Next consider the model $(\mathcal{M}^G \otimes \mathsf{M}^G)$. As $\mathcal{M}^G$ is an $\mathcal{L}_{\mathsf{EMEDL}}$-model, $(\mathcal{M}^G \otimes \mathsf{M}^G)$ is an instance of $(\mathcal{M}^G \otimes \mathsf{M}^G)$ defined in Definition 21.

Fact 3 ($\mathcal{M}^G \otimes \mathsf{M}^G$) The model obtained by updating $\mathcal{M}^G$ with M^G is a tuple

$$\mathcal{M}^G \otimes \mathsf{M}^G = (W^\otimes, E^\otimes, D^\otimes, O^\otimes, V^\otimes)$$

satisfying the following conditions.

(i) $W^\otimes = \{(w, \mathsf{s}) \in W^G \times \mathsf{S}^G \mid \mathcal{M}^G, w \models_{\mathsf{EMEDL}} \mathsf{pre}^G(\mathsf{s})\}$,

(ii) $E^{\otimes} = \{ \sim_i^{\otimes} \mid i \in A\}$ such that $(w, \mathsf{s}) \sim_i^{\otimes} (v, \mathsf{t})$ iff $w \sim_i v$ and $\mathsf{s} \sim_i^{G} \mathsf{t}$ for any $(w, \mathsf{s}), (v, \mathsf{t}) \in W^{\otimes}$,

(iii) $D^{\otimes} = \{ D^{\otimes}_{(i,j,k)} \subseteq W^{\otimes} \times W^{\otimes} \mid i, j, k \in A\}$ such that

(a) $D^{\otimes}_{(a,c,a)} = \{((w, \mathsf{s}), (v, \mathsf{s})) \in W^{\otimes} \times W^{\otimes} \mid (w, v) \in D^{G}_{(a,c,a)}\}$, and

(b) $D^{\otimes}_{(a,c,c)} = \{((w, \mathsf{s}), (v, \mathsf{s})) \in W^{\otimes} \times W^{\otimes} \mid (w, v) \in D^{G}_{(a,c,c)}\}$,

(iv) $O^{\otimes}$ is a function that assigns a subset $O^{\otimes}(i, j, h, \varphi)$ of $W^{\otimes}$ to each $(i, j, h, \varphi) \in A \times A \times H \times S_{\mathsf{EDAM^-}}$ such that $O^{\otimes}(i, j, h, \varphi) =$

(a) $(O^{G}(i, j, h, \varphi) \times \{\mathsf{S}^{G}\}) \cap W^{\otimes}$ if $\varphi \in S_{\mathsf{EMEDL}}$, and

(b) $\varnothing$ if otherwise, and

(v) $V^{\otimes}$ is a function that assigns a subset $V^{\otimes}(p)$ of $W^{\otimes}$ to each proposition letter $p \in \mathsf{P}$ such that $V^{\otimes}(p) = \{(w, \mathsf{s}) \in W^{\otimes} \mid w \in V^{G}(p)\}$. □

Since $\mathsf{S}^{G} = \{\mathsf{gl}\}$, $W^{\otimes} = \{(w, \mathsf{gl}) \mid w \in W^{G}$ and $\mathcal{M}^{G}, w \vDash_{\mathsf{EDAM^-}} \mathsf{pre}^{G}(\mathsf{s})\}$, and for any $\mathsf{s}, \mathsf{t} \in \mathsf{S}^{G}$, $\mathsf{s} = \mathsf{t} = \mathsf{gl}$. Since (35) means that $\mathcal{M}^{G}, w_1 \nvDash_{\mathsf{EDAM^-}} \mathsf{pre}^{G}(\mathsf{gl})$, and for any w such that $w \neq w_1$, (36) means that $\mathcal{M}^{G}, w \vDash_{\mathsf{EDAM^-}} \mathsf{pre}^{G}(\mathsf{gl})$, the update with $(\mathsf{M}^{G}, \mathsf{gl})$ only deletes w_1. Thus, $W^{\otimes} = (W^{G} \setminus \{w_1\}) \times \{\mathsf{gl}\}$.

Now, it is not difficult, though slightly laborious, to verify the following. Let $i \in \{a, c\}$.

$$\mathcal{M}^{G} \otimes \mathsf{M}^{G}, (w_0, \mathsf{gl}) \vDash_{\mathsf{EDAM^-}} \neg\mathsf{K}_i r \wedge \neg\mathsf{K}_i\neg r. \tag{48}$$

$$\mathcal{M}^{G} \otimes \mathsf{M}^{G}, (w_0, \mathsf{gl}) \vDash_{\mathsf{EDAM^-}} \mathsf{K}_a\neg(r \wedge \neg s). \tag{49}$$

$$\mathcal{M}^{G} \otimes \mathsf{M}^{G}, (w_0, \mathsf{gl}) \vDash_{\mathsf{EDAM^-}} \mathsf{K}_c\neg(r \wedge \neg s). \tag{50}$$

$$\mathcal{M}^{G} \otimes \mathsf{M}^{G}, (w_0, \mathsf{gl}) \vDash_{\mathsf{EDAM^-}} \neg\mathsf{O}_{(a,c,a)} r. \tag{51}$$

$$\mathcal{M}^{G} \otimes \mathsf{M}^{G}, (w_0, \mathsf{gl}) \vDash_{\mathsf{EDAM^-}} \neg(\mathsf{K}_c\mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)} r). \tag{52}$$

$$\mathcal{M}^{G} \otimes \mathsf{M}^{G}, (w_0, \mathsf{gl}) \vDash_{\mathsf{EDAM^-}} \neg\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)} r). \tag{53}$$

$$\mathcal{M}^{G} \otimes \mathsf{M}^{G}, (w_0, \mathsf{gl}) \vDash_{\mathsf{EDAM^-}} \neg\mathsf{K}_i\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)} r). \tag{54}$$

(54) means that uptake is not secured, but (49) means that a learns that $\neg(r \wedge \neg s)$. Thus, (48)–(54) show that $(\mathcal{M}^{G} \otimes \mathsf{M}^{G}, (w_0, \mathsf{gl}))$ nicely represents what has actually been done.

The guru's final utterance can also be treated in a similar way. As it can be considered as an act of commanding of the form $\mathrm{Command}_{(c,a)} r$, it can be denoted by com. Then, let $\mathsf{M}^{C} = (\mathsf{S}^{C}, \{\sim_a^{C}, \sim_c^{C}\}, \mathsf{pre}^{C})$ be a deontic action model such that (i) $\mathsf{S}^{C} = \{\mathsf{com}\}$, (ii) $\sim_a^{C} = \sim_c^{C} = \{(\mathsf{com}, \mathsf{com})\}$, and (iii) $\mathsf{pre}^{C}(\mathsf{com}) = \mathsf{Auth}(c, a, \mathrm{g})r$. Then, her acts of commanding can be represented by $(\mathsf{M}^{C}, \mathsf{com})$.

Since $(\mathsf{M}^C, \mathsf{com})$ is a pointed deontic action model, it is an instance of (M, s) in Definition 15. As $\mathcal{M}^G$ is an $\mathcal{L}_{\mathsf{EMEDL}}$-model, $(\mathcal{M}^G \otimes \mathsf{M}^C, (w_0, \mathsf{com}))$ can be used to represent the final situation in Example 1 in DAM^-, and it is not difficult to verify the following.

$$\mathcal{M}^G \otimes \mathsf{M}^C, (w_0, \mathsf{com}) \vDash_{\mathsf{DAM}^-} \mathsf{O}_{(a,c,c)}r. \tag{55}$$
$$\mathcal{M}^G \otimes \mathsf{M}^C, (w_0, \mathsf{com}) \vDash_{\mathsf{DAM}^-} \mathsf{K}_a\mathsf{O}_{(a,c,c)}r. \tag{56}$$
$$\mathcal{M}^G \otimes \mathsf{M}^C, (w_0, \mathsf{com}) \vDash_{\mathsf{DAM}^-} \mathsf{K}_c\mathsf{O}_{(a,c,c)}r. \tag{57}$$

Thus, uptake is finally secured.

One important thing about Example 1 is the fact that c not only had the suitable authority for performing an act of commanding of the form $\mathrm{Command}_{(c,a)}r$ but also was in a position to make a request of the form $\mathrm{Request}_{(c,a)}r$. Thus, it was up to her to decide, in that situation, whether to issue a command of the form $\mathrm{Command}_{(c,a)}r$ or to make a request of the form $\mathrm{Request}_{(c,a)}r$. Although what illocutionary act an utterance counts as in the context in which it is made is not just a matter of the intention of the utterer, it was the unclarity of c's intention that made a uncertain whether he was commanded or requested when she said "I would be happy if you could lead our group in that demonstration." The availability of explicit performative formulas is, obviously, an important practical solution to such a problem of uncertainty.

6. Concluding Remarks

We have developed a method of deontic product update by combining the idea of the (epistemic) product update developed in Baltag, Moss, & Solecki (1998) and the idea of the deontic update of the dynamic logic of acts of commanding, promising, and requesting developed in Yamada (2011, 2016).

We then applied it to two scenarios in which some or all of the agents involved are not certain whether a command is given or a request is made. In the case of the scenario in which the addressee understands that a request is made but another agent does not, the deontic product update worked fine, but in the case of the other scenario in which the securing of uptake fails, we have seen that the deontic product update is inapplicable. Since the securing of uptake (i.e., bringing about the understanding of the meaning and the force of the locution) is necessary for performing illocutionary acts, neither an act of commanding nor an act of requesting has been performed in this scenario. As we have also seen, however, the addressee learns that the guru would be happy if the addressee could lead the group in the demonstration in this scenario, and the epistemic product update is applicable to what has happened in such a case. Moreover, we have also seen that by making the illocutionary force explicit, uptake is secured in the end in this example.

In ordinary situations, indirectness is often very important, but it may sometimes make the securing of uptake uncertain. In such cases, the epistemic product update and the deontic product update may be used to capture different (un)certainties about different aspects of the situations and analyze how they affect the outcome.

Appendix: The three-option analysis of requesting

In this appendix, we report our recent finding about acts of requesting: the deontic product update based on the three-option analysis of acts of requesting presented in Yamada (2011) fails to capture *d*'s uncertainty in Example 3. As we have seen, acts of requesting allow for the option of refusal. The two-option analysis of Yamada (2016) captures this characteristic of requesting in terms of what the requestee needs to do in response to a request. She is not obligated to do what is requested, but she has to respond. A positive response amounts to agreeing to do what is requested and a negative response amounts to refusing to do it. These are the two options captured by the two-option analysis of requesting. When what is requested can be done on the spot, however, the requestee might do it without saying anything. The three-option analysis counts such a response as another option distinct from the above two options. It can be incorporated into the semantics of $\mathcal{L}_{\mathsf{DMEDL}}$ by replacing the clause (iii) of Definition 11 *mutatis mutandis* with the following.

(iii^3) $\mathcal{M}_{\mathrm{Request}_{(i,j)}\psi}$ is the $\mathcal{L}_{\mathsf{MEDL}}$-model obtained from $\mathcal{M}$ by replacing $D^{\mathcal{M}}(j,i,i)$ with $\{(s,t) \in D^{\mathcal{M}}(j,i,i) \mid \mathcal{M},t \vDash_{\mathsf{DMEDL}^3} (\psi \vee \mathsf{K}_i\mathsf{O}_{(j.i.j)}\psi \vee \mathsf{K}_i\neg\mathsf{O}_{(j.i.j)}\psi)\}$.

This validates the following variant of (RUGO).

Proposition 4 (Yamada 2011, p. 75.) The following principle holds.

(RUGO^3) If φ is free of occurrences of $\mathsf{O}_{(j,i,i)}$,
$[\mathrm{Request}_{(i,j)}\varphi]\mathsf{O}_{(j,i,i)}(\varphi \vee \mathsf{K}_i\mathsf{O}_{(j.i.j)}\varphi \vee \mathsf{K}_i\neg\mathsf{O}_{(j.i.j)}\varphi)$ is valid. □

As Yamada (2016, p. 485ff) only says that the simpler formulation is preferred "for a technical reason" in presenting the two-option analysis, the following discussions may be of some significance.[21]

Now we define a three-option variant of the satisfaction relation $\vDash_{\mathsf{DAM}^{-3}}$ for $\mathcal{L}_{\mathsf{DAM}^{-}}$.

Definition 23 ($\vDash_{\mathsf{DAM}^{-3}}$) Given the language $\mathcal{L}_{\mathsf{DAM}^{-}}$ based on the same countably infinite set of proposition letters P with $r \in \mathsf{P}$, the same finite set of agents $A = \{a, c, d\}$, the same finite set of organizations H with $g \in H$, and the action model M^R, let $\mathcal{M}$ be an $\mathcal{L}_{\mathsf{EMEDL}}$-model (W, E, D, O, V) such that $E = \{\sim_i^{\mathcal{M}} \mid i \in A\}$ is a set of equivalence relations over $W \times W$, $D = \{D_{(i,j,k)}^{\mathcal{M}} \mid i, j, k \in A\}$, and $O(c, a, \mathrm{g}, r) = W$. If $w \in W$, $\mathsf{s} \in \mathsf{S}^R$, $p \in \mathsf{P}$, $\varphi, \psi \in S_{\mathsf{DAM}^{-}}$, $i, j, k \in A$, and $h \in H$, then the satisfaction relation $\vDash_{\mathsf{DAM}^{-3}}$ is defined as follows.

(a) $\mathcal{M}, w \vDash_{\mathsf{DAM}^{-3}} p$ iff $w \in V(p)$,
(b) $\mathcal{M}, w \vDash_{\mathsf{DAM}^{-3}} \neg\varphi$ iff $\mathcal{M}, w \nvDash_{\mathsf{DAM}^{-3}} \varphi$,
(c) $\mathcal{M}, w \vDash_{\mathsf{DAM}^{-3}} (\varphi \wedge \psi)$ iff $\mathcal{M}, w \vDash_{\mathsf{DAM}^{-3}} \varphi$ and $\mathcal{M}, w \vDash_{\mathsf{DAM}^{-3}} \psi$,
(d) $\mathcal{M}, w \vDash_{\mathsf{DAM}^{-3}} \mathsf{K}_i\varphi$ iff for every v such that $w \sim_i^{\mathcal{M}} v$, $\mathcal{M}, v \vDash_{\mathsf{DAM}^{-3}} \varphi$,
(e) $\mathcal{M}, w \vDash_{\mathsf{DAM}^{-3}} \mathsf{O}_{(i,j,k)}\varphi$ iff for every v such that $(w, v) \in D_{(i,j,k)}^{\mathcal{M}}$, $\mathcal{M}, v \vDash_{\mathsf{DAM}^{-3}} \varphi$,
(f) $\mathcal{M}, w \vDash_{\mathsf{DAM}^{-3}} \mathsf{Auth}(i, j, h)\varphi$ iff $\varphi \in S_{\mathsf{EMEDL}}$ and $w \in O(i, j, h, \varphi)$, and
(g) $\mathcal{M}, w \vDash_{\mathsf{DAM}^{-3}} [(\mathsf{M}^R, \mathsf{s})]\varphi$ iff $\mathcal{M}, w \vDash_{\mathsf{DAM}^{-3}} \mathsf{pre}^R(\mathsf{s})$ implies $\mathcal{M} \otimes \mathsf{M}^R, (w, \mathsf{s}) \vDash_{\mathsf{DAM}^{-3}} \varphi$,

where $\mathcal{M} \otimes \mathsf{M}^R$ is a tuple

$$(W^{\otimes}, E^{\otimes}, D^{\otimes}, O^{\otimes}, V^{\otimes})$$

satisfying the following conditions.

(i) $W^{\otimes} = \{(w, \mathsf{s}) \in W \times \mathsf{S}^R \mid \mathcal{M}, w \vDash_{\mathsf{DAM}^{-3}} \mathsf{pre}^R(\mathsf{s})\}$,
(ii) $E^{\otimes} = \{\sim_i^{\otimes} \mid i \in A\}$ such that
$(w, \mathsf{s}) \sim_i^{\otimes} (v, \mathsf{t})$ iff $w \sim_i^{\mathcal{M}} v$ and $\mathsf{s} \sim_i^R \mathsf{t}$ for any $(w, \mathsf{s}), (v, \mathsf{t}) \in W^{\otimes}$,
(iii) $D^{\otimes} = \{D_{(i,j,k)}^{\otimes} \mid, i, j, k \in A\}$ such that
$D_{(i,j,k)}^{\otimes} = ((w, \mathsf{s}), (v, \mathsf{t})) \in W^{\otimes} \times W^{\otimes} \mid (w, v) \in D_{(i,j,k)}^{\mathsf{s}}$ and $\mathsf{s} = \mathsf{t}\}$
where $D_{(i,j,k)}^{\mathsf{s}} =$

[21] The difficulty discussed in this appendix was not known when I wrote Yamada (2016). The "technical reason" I had in mind in writing Yamada (2016) was just the simplicity of the analysis. Note that this footnote is added after the manuscript of this paper was accepted for publication.

(a) $D^{\mathcal{M}}_{(i,j,k)}$ if $(i, j, k) \neq (a, c, c)$,

(b) $\{(w,v) \in D^{\mathcal{M}}_{(a,c,c)} \mid \mathcal{M}, v \vDash_{\mathsf{DAM}^{-3}} r\}$ if $\mathsf{s} = \mathsf{com}$, and

(c) $\{(w,v) \in D^{\mathcal{M}}_{(a,c,c)} \mid \mathcal{M}, v \vDash_{\mathsf{DAM}^{-3}} (r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)\}$ if $\mathsf{s} = \mathsf{req}$,

(iv) $O^{\otimes}$ is a function that assigns a subset $O^{\otimes}(i, j, h, \varphi)$ of $W^{\otimes}$ to each $(i, j, h, \varphi) \in A \times A \times H \times S_{\mathsf{DAM}^{-}}$ such that $O^{\otimes}(i, j, h, \varphi) =$

(a) $(O(i, j, h, \varphi) \times \mathsf{S}^R) \cap W^{\otimes}$ if $\varphi \in S_{\mathsf{EMEDL}}$, and

(b) $\varnothing$ if otherwise, and

(v) $V^{\otimes}$ is the function that assigns a subset $V^{\otimes}(p)$ of $W^{\otimes}$ to each proposition letter $p \in \mathsf{P}$ such that $V^{\otimes}(p) = \{(w, \mathsf{s}) \in W^{\otimes} \mid w \in V(p)\}$. □

M^R is the action model defined in Definition 16 (pp. 20–21). $\mathcal{M} \otimes \mathsf{M}^R$ is the result of updating $\mathcal{M}$ with M^R. As Clauses (iii)-(b) and (iii)-(c) are based on the clauses defining $\mathcal{M}_{\mathrm{Command}_{(i,j)}\psi}$ and $\mathcal{M}_{\mathrm{Request}_{(i,j)}\psi}$ in the semantics of DMEDL based on the three-option analysis, respectively, $\mathcal{L}_{\mathsf{DAM}^{-}}$ with $(\mathsf{M}^R, \mathsf{s})$ added is not a universal language for dealing with illocutionary acts in general but a specific language for dealing with com and req.

For the sake of safety, we require (1) φ to be in S_{EMEDL} in Clause (f) and (2) $O^{\otimes}(i, j, h, \varphi)$ to be the empty set when $\varphi \notin S_{\mathsf{EMEDL}}$. (1) and (2) falsify $\mathsf{Auth}_{(i,j,h)}\varphi$ when $\varphi \notin S_{\mathsf{EMEDL}}$. As S^R = $\{\mathsf{com}, \mathsf{req}\}$, for any $\mathsf{t} \in \mathsf{S}^R$, $\mathsf{pre}(\mathsf{t}) \in S_{\mathsf{EMEDL}}$.

As $\sim^{\mathcal{M}}_i$ and $\sim^R_i$ are equivalence relations, $\sim^{\otimes}_i$ is an equivalence relation. Therefore, $\mathcal{M} \otimes \mathsf{M}^R$ is an $\mathcal{L}_{\mathsf{EMEDL}}$-model.

In order to analyze Example 3, we need a model suitable for representing the initial situation of the event $(\mathsf{M}^R, \mathsf{req})$. The model $(\mathcal{M}^R, w_0)$ defined in Section 4 seems to be suitable. By arguments similar to those in Section 4, it is not difficult to confirm the following statements. Let $i \in \{a, c, d\}$.

$$\mathcal{M}^R, w_0 \vDash_{\mathsf{DAM}^{-3}} \neg\mathsf{K}_i r \wedge \neg\mathsf{K}_i\neg r. \tag{58}$$

$$\mathcal{M}^R, w_0 \vDash_{\mathsf{DAM}^{-3}} \neg\mathsf{O}_{(a,c,a)}r \wedge \neg\mathsf{O}_{(a,c,a)}\neg r. \tag{59}$$

$$\mathcal{M}^R, w_0 \vDash_{\mathsf{DAM}^{-3}} \neg\mathsf{O}_{(a,c,c)}r \wedge \neg\mathsf{O}_{(a,c,c)}\neg r. \tag{60}$$

$$\mathcal{M}^R, w_0 \vDash_{\mathsf{DAM}^{-3}} \neg\mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{61}$$

If $(\mathcal{M}^R, w_0)$ is really suitable for representing how $(\mathsf{M}^R, \mathsf{req})$ works, however, we must also have the following.

$$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{62}$$

$$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \mathsf{K}_a\mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{63}$$

$$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \mathsf{K}_c \mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r). \tag{64}$$

$$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \neg \mathsf{K}_d \mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r). \tag{65}$$

They are the three-option analogues of (5)–(8). (62) means that it is obligatory for a towards c due to c to see to it that $r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r$ in the final situation of $(\mathsf{M}^R, \mathsf{req})$. Thus, (63), (64), and (65) jointly mean that a and c, but not d, know what has happened in the final situation of $(\mathsf{M}^R, \mathsf{req})$. So, if (63), (64), and (65) are shown to hold, we can say that uptake is secured although d remains uncertain.

It is not difficult to prove (62), (63), and (64), but (65) is disproved. We now show more generally that $\neg \mathsf{K}_d \mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r)$ is not satisfiable in any pointed $\mathcal{L}_{\mathsf{EMEDL}}$-model $(\mathcal{M} \otimes \mathsf{M}^R, (w, \mathsf{req}))$ based on the sets P, A, and H. Note that we can assume that $O(c, a, \mathrm{g}, r) = W$ without loss of generality, as we will argue, in the relevant part of the proof, by showing that for any $(v, \mathsf{com}) \in W^{\otimes}$ that is $\sim_d^{\otimes}$ -accessible from (w, req), $\mathcal{M} \otimes \mathsf{M}^R, (v, \mathsf{com}) \vDash_{\mathsf{DAM}^{-3}} \mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r)$.

Fact 4 (The difficulty) Let the language $\mathcal{L}_{\mathsf{DAM}^-}$ based on the same countably infinite set of proposition letters P, the same finite set of agents $A = \{a, c, d\}$, and the same finite set of organizations H such that $\mathrm{g} \in H$ be given. Then, for any $\mathcal{L}_{\mathsf{EMEDL}}$-model $\mathcal{M} = (W, E, D, O, V)$ and a world w of $\mathcal{M}$ such that $E = \{\sim_i^{\mathcal{M}} \mid i \in A\}$, $D = \{D^{\mathcal{M}}_{(i,j,k)} \mid i, j, k \in A\}$, and $O(c, a, \mathrm{g}, r) = W$,

$$\mathcal{M} \otimes \mathsf{M}^R, (w, \mathsf{req}) \nvDash_{\mathsf{DAM}^{-3}} \neg \mathsf{K}_d \mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r).$$

[Proof] Let the language $\mathcal{L}_{\mathsf{DAM}^-}$ based on the same countably infinite set of proposition letters P, the same finite set of agents $A = \{a, c, d\}$, and the same finite set of organizations H such that $\mathrm{g} \in H$ be given. Fix an arbitrary $\mathcal{L}_{\mathsf{EMEDL}}$-model $\mathcal{M} = (W, E, D, O, V)$ and a world w of $\mathcal{M}$ such that $E = \{\sim_i^{\mathcal{M}} \mid i \in A\}$, $D = \{D^{\mathcal{M}}_{(i,j,k)} \mid i, j, k \in A\}$, and $O(c, a, \mathrm{g}, r) = W$. Then, consider the result $\mathcal{M} \otimes \mathsf{M}^R = (W^{\otimes}, E^{\otimes}, D^{\otimes}, O^{\otimes}, V^{\otimes})$ of updating $\mathcal{M}$ with M^R. Since $\mathsf{pre}(\mathsf{req}) = \top$, $(w, \mathsf{req}) \in W^{\otimes}$, and so, we can work with a pointed model $(\mathcal{M} \otimes \mathsf{M}^R, (w, \mathsf{req}))$. Since $\mathsf{com} \sim_d^R \mathsf{req}$ in M^R,

$$\mathcal{M} \otimes \mathsf{M}^R, (w, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \neg \mathsf{K}_d \mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r)$$

iff either (a) for some (v, req) such that $(w, \mathsf{req}) \sim_d^{\otimes} (v, \mathsf{req})$,

$$\mathcal{M} \otimes \mathsf{M}^R, (v, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \neg \mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r)$$

or (b) for some (v, com) such that $(w, \mathsf{req}) \sim_d^{\otimes} (v, \mathsf{com})$,

$$\mathcal{M} \otimes \mathsf{M}^R, (v, \mathsf{com}) \vDash_{\mathsf{DAM}^{-3}} \neg \mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r).$$

We show that neither (a) nor (b) can be true. First, consider (a). For any world v of $\mathcal{M}$, we have

$$\mathcal{M} \otimes \mathsf{M}^R, (v, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \neg \mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r)$$

iff for some world (u, req) of $\mathcal{M} \otimes \mathsf{M}^R$ such that $((v, \mathsf{req}), (u, \mathsf{req})) \in D^{\otimes}_{(a,c,c)}$,

$$\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \neg(r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r).$$

Since $\mathsf{req} \in \mathsf{S}^R$ and $\mathsf{req} = \mathsf{req}$, however, for any worlds v and u of $\mathcal{M}$,

$$((v, \mathsf{req}), (u, \mathsf{req})) \in D^{\otimes}_{(a,c,c)}$$

iff $(v, u) \in D^{\mathsf{req}}_{(a,c,c)}$

iff $(v, u) \in D^{\mathcal{M}}_{(a,c,c)}$ and $\mathcal{M}, u \vDash_{\mathsf{DAM}^{-3}} (r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r)$.

Note that for any world u of $\mathcal{M}$, we have

$$\mathcal{M}, u \vDash_{\mathsf{DAM}^{-3}} (r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r)$$

iff $\mathcal{M}, u \vDash_{\mathsf{DAM}^{-3}} r$, or $\mathcal{M}, u \vDash_{\mathsf{DAM}^{-3}} \mathsf{K}_c \mathsf{O}_{(a,c,a)} r$, or $\mathcal{M}, u \vDash_{\mathsf{DAM}^{-3}} \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r$.

But for any world u of $\mathcal{M}$ again, we have

$$\mathcal{M}, u \vDash_{\mathsf{DAM}^{-3}} r$$

iff $u \in V(r)$

iff $(u, \mathsf{req}) \in V^{\otimes}(r)$

iff $\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} r$,

$$\mathcal{M}, u \vDash_{\mathsf{DAM}^{-3}} \mathsf{K}_c \mathsf{O}_{(a,c,a)} r$$

iff for any t such that $u \sim_c^{\mathcal{M}} t$, $\mathcal{M}, t \vDash_{\mathsf{DAM}^{-3}} \mathsf{O}_{(a,c,a)} r$

iff for any t such that $u \sim_c^{\mathcal{M}} t$ for any s such that $(t, s) \in D^{\mathcal{M}}_{(a,c,a)}$, $\mathcal{M}, s \vDash_{\mathsf{DAM}^{-3}} r$

iff for any t such that $u \sim_c^{\mathcal{M}} t$ for any s such that $(t, s) \in D^{\mathcal{M}}_{(a,c,a)}$,

$$\mathcal{M} \otimes \mathsf{M}^R, (s, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} r$$

iff for any t such that $u \sim_c^{\mathcal{M}} t$ for any (s, req) such that $((t, \mathsf{req}), (s, \mathsf{req})) \in D^{\otimes}_{(a,c,a)}$,

$$\mathcal{M} \otimes \mathsf{M}^R, (s, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} r$$

iff for any t such that $u \sim_c^{\mathcal{M}} t$, $\mathcal{M} \otimes \mathsf{M}^R, (t, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \mathsf{O}_{(a,c,a)} r$

iff for any (t, req) such that $(u, \mathsf{req}) \sim_c^{\otimes} (t, \mathsf{req})$, $\mathcal{M} \otimes \mathsf{M}^R, (t, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \mathsf{O}_{(a,c,a)} r$
iff $\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \mathsf{K}_c \mathsf{O}_{(a,c,a)} r$,

and

$\mathcal{M}, u \vDash_{\mathsf{DAM}^{-3}} \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r$
iff for any t such that $u \sim_c^{\mathcal{M}} t$, $\mathcal{M}, t \vDash_{\mathsf{DAM}^{-3}} \neg \mathsf{O}_{(a,c,a)} r$
iff for any t such that $u \sim_c^{\mathcal{M}} t$ for some s such that $(t, s) \in D_{(a,c,a)}^{\mathcal{M}}$, $\mathcal{M}, s \vDash_{\mathsf{DAM}^{-3}} \neg r$
iff for any t such that $u \sim_c^{\mathcal{M}} t$ for some s such that $(t, s) \in D_{(a,c,a)}^{\mathcal{M}}$,
$\mathcal{M} \otimes \mathsf{M}^R, (s, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \neg r$
iff for any t such that $u \sim_c^{\mathcal{M}} t$ for some (s, req) such that $((t, \mathsf{req}), (s, \mathsf{req})) \in D_{(a,c,a)}^{\otimes}$,
$\mathcal{M} \otimes \mathsf{M}^R, (s, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \neg r$
iff for any t such that $u \sim_c^{\mathcal{M}} t$, $\mathcal{M} \otimes \mathsf{M}^R, (t, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \neg \mathsf{O}_{(a,c,a)} r$
iff for any (t, req) such that $(u, \mathsf{req}) \sim^{\otimes} (t, \mathsf{req})$, $\mathcal{M} \otimes \mathsf{M}^R, (t, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \neg \mathsf{O}_{(a,c,a)} r$
iff $\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r$.

Thus, we have

For any world u of $\mathcal{M}$, $\mathcal{M}, u \vDash_{\mathsf{DAM}^{-3}} (r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r)$
iff $\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} (r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r)$.

This means we have

For any worlds v and u of $\mathcal{M}$, $((v, \mathsf{req}), (u, \mathsf{req})) \in D_{(a,c,c)}^{\otimes}$
iff $(v, u) \in D_{(a,c,c)}^{\mathcal{M}}$ and $\mathcal{M}, u \vDash_{\mathsf{DAM}^{-3}} (r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r)$
iff $(v,u) \in D_{(a,c,c)}^{\mathcal{M}}$ and $\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} (r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r)$.

This, in turn, means that for any world v of $\mathcal{M}$,

$\mathcal{M} \otimes \mathsf{M}^R, (v, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \neg \mathsf{O}_{(a,c,c)} (r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r)$
iff for some world (u, req) of $\mathcal{M} \otimes \mathsf{M}^R$ such that $((v, \mathsf{req}), (u, \mathsf{req})) \in D_{(a,c,c)}^{\otimes}$,
$\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \neg (r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r)$
iff for some world (u, req) of $\mathcal{M} \otimes \mathsf{M}^R$ such that $(v, u) \in D_{(a,c,c)}^{\mathcal{M}}$,
$\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} (r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r)$ and
$\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \neg (r \vee \mathsf{K}_c \mathsf{O}_{(a,c,a)} r \vee \mathsf{K}_c \neg \mathsf{O}_{(a,c,a)} r)$

iff for some world (u, req) of $\mathcal{M} \otimes \mathsf{M}^R$ such that $(v, u) \in D^{\mathcal{M}}_{(a,c,c)}$,

$$\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} (r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r) \wedge \neg(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r).$$

This, of course, is impossible, and so, (a) cannot be true.

Then, consider (b). As $O(c, a, \mathrm{g}, r) = W$, $O^{\otimes}(c, a, \mathrm{g}, r) = W \times \mathsf{S}^R$ and $W^{\otimes} = (W \times \{\mathsf{com}\}) \cup (W \times \{\mathsf{req}\})$.

Now, for any world v of $\mathcal{M}$,

$$\mathcal{M} \otimes \mathsf{M}^R, (v, \mathsf{com}) \vDash_{\mathsf{DAM}^{-3}} \neg\mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$$

iff for some world (u, com) of $\mathcal{M} \otimes \mathsf{M}^R$ such that $((v, \mathsf{com}), (u, \mathsf{com})) \in D^{\otimes}_{(a,c,c)}$,

$$\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{com}) \vDash_{\mathsf{DAM}^{-3}} \neg(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r).$$

Since $\mathsf{com} \in \mathsf{S}^R$ and $\mathsf{com} = \mathsf{com}$, however, for any worlds v and u of $\mathcal{M}$,

$$((v, \mathsf{com}), (u, \mathsf{com})) \in D^{\otimes}_{(a,c,c)}$$

iff $(v, u) \in D^{\mathsf{com}}_{(a,c,c)}$

iff $(v, u) \in D^{\mathcal{M}}_{(a,c,c)}$ and $\mathcal{M}, u \vDash_{\mathsf{DAM}^{-3}} r$.

Now, by a reasoning similar to the one in the case of (a), for any world u of $\mathcal{M}$, we can establish

$$\mathcal{M}, u \vDash_{\mathsf{DAM}^{-3}} r \text{ iff } \mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{com}) \vDash_{\mathsf{DAM}^{-3}} r.$$

This, in turn, means that for any world v of $\mathcal{M}$, we have

$$\mathcal{M} \otimes \mathsf{M}^R, (v, \mathsf{com}) \vDash_{\mathsf{DAM}^{-3}} \neg\mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$$

iff for some world (u, com) of $\mathcal{M} \otimes \mathsf{M}^R$ such that $((v, \mathsf{com}), (u, \mathsf{com})) \in D^{\otimes}_{(a,c,c)}$,

$$\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{com}) \vDash_{\mathsf{DAM}^{-3}} \neg(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$$

iff for some world (u, com) of $\mathcal{M} \otimes \mathsf{M}^R$

such that $(v, u) \in D^{\mathcal{M}}_{(a,c,c)}$ and $\mathcal{M}, u \vDash_{\mathsf{DAM}^{-3}} r$,

$$\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{com}) \vDash_{\mathsf{DAM}^{-3}} \neg(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$$

iff for some world (u, com) of $\mathcal{M} \otimes \mathsf{M}^R$ such that $(v, u) \in D^{\mathcal{M}}_{(a,c,c)}$,

$\mathcal{M}, u \vDash_{\mathsf{DAM}^{-3}} r$ and

$$\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{com}) \vDash_{\mathsf{DAM}^{-3}} \neg(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$$

iff for some world (u, com) of $\mathcal{M} \otimes \mathsf{M}^R$ such that $(v, u) \in D^{\mathcal{M}}_{(a,c,c)}$,

$$\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{com}) \vDash_{\mathsf{DAM}^{-3}} r \text{ and}$$
$$\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{com}) \vDash_{\mathsf{DAM}^{-3}} \neg(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$$
iff for some world (u, com) of $\mathcal{M} \otimes \mathsf{M}^R$ such that $(v, u) \in D^{\mathcal{M}}_{(a,c,c)}$,
$$\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{com}) \vDash_{\mathsf{DAM}^{-3}} r \wedge \neg(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)\,.$$

This, again, is impossible. Thus, neither (a) nor (b) can be true, and so, $\neg\mathsf{k}_d(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$ cannot be satisfied in the situation represented by ($\mathcal{M} \otimes \mathsf{M}^R$, (w, req)). □

The following two points are crucial here.

1. In the three-option analysis of acts of requesting, an act of requesting of the form Request$_{(a,c)}$ r cuts every link of deontic accessibility for (a, c, c) that arrives in a world where $r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r$ does not hold. Thus, for any worlds w and u of $\mathcal{M}$, $((w, \mathsf{req}), (v, \mathsf{req})) \in D^{\otimes}_{(a,c,c)}$ implies $\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} (r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$. This, in turn, implies that for any world w of $\mathcal{M}$, $\mathcal{M} \otimes \mathsf{M}^R$, $(w, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$.
2. An act of commanding of the form Command$_{(a,c)}r$, on the other hand, cuts every link of deontic accessibility for (a, c, c) that arrives in a world where r does not hold. Thus, for any worlds w and u of $\mathcal{M}$, $((w, \mathsf{com}), (v, \mathsf{com})) \in$ implies $\mathcal{M} \otimes \mathsf{M}^R, (u, \mathsf{com}) \vDash_{\mathsf{DAM}^{-3}} r$. This implies that for any world w of $\mathcal{M}$, $\mathcal{M} \otimes \mathsf{M}^R$, $(w, \mathsf{com}) \vDash_{\mathsf{DAM}^{-3}} \mathsf{O}_{(a,c,c)}r$. Since r implies $(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$, however, this means that for any world w of $\mathcal{M}$, we have $\mathcal{M} \otimes \mathsf{M}^R$, $(w, \mathsf{com}) \vDash_{\mathsf{DAM}^{-3}} \mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$.

These two facts jointly make $\neg\mathsf{K}_d\mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$ unsatisfiable in $\mathcal{M} \otimes \mathsf{M}^R$.

A brief comparison of the three-option analysis and the two-option analysis of acts of requesting may be in order here. Consider the following principles.

(CUGO) If φ is free of occurrences of $\mathrm{O}_{(j,i,i)}$,

[Command$_{(i,j)}\varphi$]$\mathsf{O}_{(j,i,i)}\varphi$ is valid.

(ROSS^C) If φ is free of occurrences of $\mathsf{O}_{(j,i,i)}$,

[Command$_{(i,j)}\varphi$]$\mathsf{O}_{(j,i,i)}(\varphi \vee \mathsf{K}_i\mathsf{O}_{(j.i.j)}\varphi \vee \mathsf{K}_i\neg\mathsf{O}_{(j.i.j)}\varphi)$ is valid.

(RUGO^3) If φ is free of occurrences of $\mathsf{O}_{(j,i,i)}$,

[Request$_{(i,j)}\varphi$]$\mathsf{O}_{(j,i,i)}(\varphi \vee \mathsf{K}_i\mathsf{O}_{(j.i.j)}\varphi \vee \mathsf{K}_i\neg\mathsf{O}_{(j.i.j)}\varphi)$ is valid.

(RUGO^2) If φ is free of occurrences of $\mathsf{O}_{(j,i,i)}$,

$[\mathrm{Request}_{(i,j)}\varphi]\mathsf{O}_{(j,i,i)}(\mathsf{K}_i\mathsf{O}_{(j.i.j)}\varphi \vee \mathsf{K}_i\neg\mathsf{O}_{(j.i.j)}\varphi)$ is valid.

(ROSS^{R2}) If φ is free of occurrences of $\mathsf{O}_{(j,i,i)}$,

$[\mathrm{Request}_{(i,j)}\varphi]\mathsf{O}_{(j,i,i)}(\varphi \vee \mathsf{K}_i\mathsf{O}_{(j.i.j)}\varphi \vee \mathsf{K}_i\neg\mathsf{O}_{(j.i.j)}\varphi)$ is valid.

(RUGO^3) and (RUGO^2) are based on the three-option analysis and the two-option analysis of acts of requesting, respectively. The fact that (CUGO) implies (ROSS^C) is a variant of the problem known as Ross Paradox (Ross 1944). (ROSS^C) and (RUGO^3) jointly show that both an act of commanding of the form $\mathrm{Command}_{(i,j)}\varphi$ and an act of requesting of the form $\mathrm{Request}_{(i,j)}\varphi$ generate the obligation of the form $\mathsf{O}_{(j,i,i)}(\varphi \vee \mathsf{K}_i\mathsf{O}_{(j.i.j)}\varphi \vee \mathsf{K}_i\neg\mathsf{O}_{(j.i.j)}\varphi)$ if φ is free of occurrences of $\mathsf{O}_{(j,i,i)}$. This means that $\mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$ holds everywhere in $\mathcal{M} \otimes \mathsf{M}^R$ in the three-option analysis, since com is an act of commanding of the form $\mathrm{Command}_{(c,a)}r$ and req is an act of requesting of the form $\mathrm{Request}_{(c,a)}r$. Thus, we do not have

$$\mathcal{M} \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^{-3}} \neg\mathsf{K}_d\mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{65*}$$

As we need (65*) in order to represent d's uncertainty in Example 3 in the three-option analysis, the three-option analysis does not work well in analyzing Example 3.[22]

As φ does not imply $(\mathsf{K}_c\mathsf{O}_{(a,c,a)}\varphi \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}\varphi)$, however, (CUGO) does not imply $[\mathrm{Command}_{(c,a)}\varphi]\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a.c.a)}\varphi \vee \mathsf{K}_c\neg\mathsf{O}_{(a.c.a)}\varphi)$. Note that ($\mathrm{RUGO}^2$) implies ($\mathrm{ROSS}^{R2}$) and that ($\mathrm{ROSS}^C$) and ($\mathrm{ROSS}^{R2}$) jointly imply that $\mathsf{O}_{(a,c,c)}(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$ holds everywhere in $\mathcal{M}^R \otimes \mathsf{M}^R$ in the two-option analysis. This, however, does not make $\neg\mathsf{K}_d\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r)$ unsatisfiable in $\mathcal{M}^R \otimes \mathsf{M}^R$, and as we have seen in Section 4, in fact, we have

$$\mathcal{M}^R \otimes \mathsf{M}^R, (w_0, \mathsf{req}) \vDash_{\mathsf{DAM}^-} \neg\mathsf{K}_d\mathsf{O}_{(a,c,c)}(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{8}$$

This is possible since

$$(r \vee \mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r) \wedge (r \wedge \neg(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r))$$

is not a contradiction. As we have seen, we have

22 Since $\mathcal{M}^R \otimes \mathsf{M}^R$ is an instance of $\mathcal{M} \otimes \mathsf{M}^R$, (65) fails, of course.

$$\mathcal{M}^R \otimes \mathsf{M}^R, (v_2, \mathsf{com}) \vDash_{\mathsf{DAM}^-} r \tag{28}$$

and

$$\mathcal{M}^R \otimes \mathsf{M}^R, (v_2, \mathsf{com}) \vDash_{\mathsf{DAM}^-} \neg(\mathsf{K}_c\mathsf{O}_{(a,c,a)}r \vee \mathsf{K}_c\neg\mathsf{O}_{(a,c,a)}r). \tag{31}$$

As (v_2, com) is $D^{\otimes}_{(a,c,c)}$-accessible from (w_0, com) and (w_0, com) is $\sim^{\otimes}_{d}$-accessible from (w_0, req), (8) holds. Thus, the deontic product update based on the two-option analysis does not suffer from the problem of that based on the three-option analysis.

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